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On a charged particle's spin evolution induced by a strong laser

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Abstract. The quantum and classical dynamics of a charged, spin-1/2 particle interacting with a strong laser, modeled by the elliptically polarized monochromatic electromagnetic plane wave, is considered in the semi-classical approach. The charge interaction with a high intensity radiation is described classically without using of the dipole approximation. The particle's spin evolution is treated according to the Pauli equation with relativistic corrections caused by the laser intensity. The resonance character of transitions between spin-1/2 states for certain regimes of intensities and values of polarization is described.

1. Introduction

The recent successes in a laser technologies envisage building of the coherent light sources that can provide electromagnetic field intensities in excess of $10^{20}\,\mathrm{W\,cm^{-2}}$ (cf. [1]). This opens a new perspectives in our studies of light-matter interactions, that are important for the fundamental physics as well as for diverse applications. As studies show (see e.g. [2] and references therein), the classical/quantum dynamics of a charged particle in a strong electromagnetic background turns out to be far from a customary view. The later are mainly based on the so-called electric dipole approximation [3]. However, when the radiation intensity is sufficiently large, several factors, such as the retardation effect, the role of the magnetic part of Heaviside-Lorentz force in distortion of a particle's classical orbit as well as the Thomas correction, cannot be ignored any more. The exit beyond the dipole approximation gives not only quantitative corrections to results, obtained for a small light intensity, but sometimes may lead to their qualitative changes. The present report aims to describe such a non-dipole effects in the the spin evolution of charged particle caused by its interaction with a high intensity laser.

Quantity characterizing the intensity of a coherent radiation interaction is given by the dimensionless laser field strength parameter:

$$\eta^2 := -2\,\frac{e^2}{m^2 c^4}\,\langle\langle\, A_\mu A^\mu \,\rangle\rangle\,, \tag{1}$$

where m and e are particle mass and electric charge, $\langle\langle\cdots\rangle\rangle$ stands for time average and A_μ is a four vector describing the laser field [4, 5]. The quantity η^2 specifies the relativistic

properties of a particle's dynamics. For small $\eta^2 \ll 1$, the dipole approximation works and it is well established that a charged particle's spin behaves adiabatically, linearly responding to the magnetic component of the electromagnetic field. When η^2 is increasing the dipole approximation breaks down, non-linear effects come in force and the adiabatic picture of the spin precession turns to be inadequate. Below, we briefly state a method allowing to describe the evolution of a spin-1/2 particle in a strong monochromatic plane wave for the "semi-relativistic interaction regime", when effects of η^2- order are not negligible any more, but still $\eta^2 < 1$ and expansions over η^2 are correct.

The paper is organized as follows. After formulation of a laser-particle interaction model a short exposition of its classical dynamics beyond the dipole approximation is given following [6]. Then, in accordance with the semi-classical approach, the particle's spin dynamics is investigated solving the Pauli equation, with effective magnetic field evaluated along the charge classical trajectory. Our studies reveal several non-dipole effects induced by a strong laser, each with non-trivial dependence on a laser's intensity and polarization. Below two of them, the deviation of spin precession frequency from the laser frequency and the resonance character of a charged particle's spin-flip transitions will be discussed.

2. The model
The laser radiation is modeled by a monochromatic elliptically polarized plane wave propagated along the z-axis. The corresponding gauge potential can be fixed as follows

$$\mathbf{A} := a\left(\varepsilon \cos(\omega_L \xi), \ \sqrt{1-\varepsilon^2} \sin(\omega_L \xi), \ 0 \right), \tag{2}$$

where $a = \mathcal{E}_0/\omega_L$, the phase $\xi := t - z/c$ and $\varepsilon \in [0,1]$ is the light polarization parameter.

We suppose that a charge carrier particle is in pure quantum state Ψ, which is decomposable in charge & spin components:

$$|\Psi\rangle = \sum_{i=0,1} \sum_{\alpha=\pm} c_{\alpha,i} |\psi_\alpha\rangle \otimes |\chi_i\rangle. \tag{3}$$

Two states $|\psi_\pm\rangle$ in (3) are linearly independent semi-classical solutions to the Schrödinger equation for a charge interacting with a laser radiation

$$\imath \hbar \frac{\mathrm{d}}{\mathrm{d}t} |\psi\rangle = -\frac{\hbar^2}{2m} \left(\boldsymbol{\nabla} - \frac{\imath e}{\hbar c} \mathbf{A}(t, \boldsymbol{x}) \right)^2 |\psi\rangle. \tag{4}$$

The semi-classical calculations imply that the state vectors $|\chi_i\rangle$, describing spin degrees in (3), satisfy the spin evolution equation:

$$\imath \hbar \frac{\mathrm{d}}{\mathrm{d}t} |\chi\rangle = \mathrm{H}(t)|\chi\rangle, \tag{5}$$

where

$$\mathrm{H}(t) = -\left(\frac{ge}{2mc} \left(\mathbf{B} - \frac{1}{c} [\boldsymbol{v} \times \mathbf{E}] \right) + \frac{1}{2c^2} [\boldsymbol{v} \times \dot{\boldsymbol{v}}] \right) \mathbf{S}. \tag{6}$$

The $\mathbf{E}$ and $\mathbf{B}$ in (6) are electric and magnetic components of a laser field evaluated along a particle's classical orbit seen in the LAB frame. The term in parenthesis is the magnetic field in the instantaneous rest frame of a charged particle, (Galilei boosted), while the last contribution in (6) is the leading relativistic Thomas precession correction [7] due to the non-vanishing curvature of a particle's trajectory. Below the spin-1/2 particle will be considered, whose spin operator is $\mathbf{S} = \hbar/2 \, \boldsymbol{\sigma}$, with the standard Pauli matrices $\boldsymbol{\sigma}$.

3. Sketch of calculations

The forthcoming computations rest on the classical non-relativistic Hamilton-Jacobi problem for spinless particle moving in the electromagnetic background (2). Due to the knowledge of the exact solution to this problem [6], one can determine both, the leading semi-classical solution to the Schrödinger equation (4) as well as calculate the effective spin-laser interaction (6).

3.1. The classical dynamics

To make formulas compact let us impose the initial condition on classical trajectory $\boldsymbol{x}(0) = 0$, and fix the frame, where the average of a particle's velocity component orthogonal to the electromagnetic wave propagation direction is zero, $\langle\langle \boldsymbol{v}_\perp \rangle\rangle = 0$. According to [6] the Hamilton-Jacobi generating function for a spinless particle traveling in an arbitrary plane wave background of the form $(\boldsymbol{A}_\perp(\xi), 0)$ reads

$$\mathcal{F}(\xi, \boldsymbol{\Pi}) \;=\; -c(mc - \Pi_z)\,\xi + c\int_0^\xi \mathrm{d}u\,\sqrt{(mc - \Pi_z)^2 + W(u, \boldsymbol{\Pi}_\perp)}\,, \qquad (7)$$

where

$$W(\xi, \boldsymbol{\Pi}_\perp) := -\frac{e^2}{c^2}\,\boldsymbol{A}_\perp^2 + 2\,\frac{e}{c}\,\boldsymbol{A}_\perp \cdot \boldsymbol{\Pi}_\perp\,. \qquad (8)$$

Here the constants Π_z and $\boldsymbol{\Pi}_\perp$ are determined from the initial value of the particle velocity. With the aid of (7) the standard calculations give the leading semiclassical wave function [1] :

$$\langle \boldsymbol{x}, t | \psi_+ \rangle = \frac{1}{\sqrt{|\partial\mathcal{F}/\partial\xi|}}\,\exp\frac{i}{\hbar}\,(\mathcal{E}t + \boldsymbol{\Pi}_\perp \cdot \mathbf{x}_\perp)\,\exp\frac{i}{\hbar}\mathcal{F}. \qquad (9)$$

Since the generating functions (7) determines a particle's trajectory as an explicit function of laboratory frame time t, the evaluation of the effective magnetic field (6) along the classical orbit becomes straightforward. A quick glance shows that in order to compute (6) it is enough to owe from [6] the expression for a particle's velocity

$$\boldsymbol{v} = c\left(-c\eta\varepsilon\,\mathrm{cn}\,(u, \mu)\,, \eta\sqrt{1 - \varepsilon^2}\,\mathrm{sn}\,(u, \mu)\,, 1 - \gamma_z\,\mathrm{dn}\,(u, \mu)\right) \qquad (10)$$

as well as the expression for its z-coordinate:

$$z(t) = ct - \frac{c}{\omega_L}\,\mathrm{am}\big(u, \mu\big)\,. \qquad (11)$$

In (10)-(11) $\mathrm{sn}(u, \mu)$, $\mathrm{cn}(u, \mu)$, $\mathrm{dn}(u, \mu)$ denote the double periodic elliptic Jacobian functions, and $\mathrm{am}(u, \mu)$ is the Jacobian amplitude function [8]. The argument, $u := \omega_L' t$, of these functions is the laboratory frame time t scaled by the non-relativistically Doppler shifted laser frequency $\omega_L' = \gamma_z\,\omega_L\,, \quad c\gamma_z = c - \upsilon_z(0)$. The modulus μ is determined by a laser's and a particle's characteristics:[2] $\gamma_z^2\,\mu^2 = (1 - 2\,\varepsilon^2)\,\eta^2$. Note that, the quarter period K of the Jacobian functions is non-linear function of the modulus μ. Therefore, the fundamental circular frequency of the particle's motion, $\omega_P = 2\pi/T_P$, differs from the frequency ω_L of a laser field. According to (10) the components of a charged particle velocity in the plane orthogonal to the wave propagation are periodic functions of time with the period $T_P := 4\mathrm{K}/\omega_L'$, while in the direction of propagation the period of oscillations is twice smaller.

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[1] To simplify expressions we assume that the initial state has only one non vanishing coefficient, $c_{+,0}$. The unit normalization condition for (9) fixes it $\pi c_{+,0}^2 = 2m\,\omega_P$.

[2] In (10)-(11) the modulus μ belongs to the fundamental domain, $0 < \mu^2 < 1$. The solutions outside this interval can be reconstructed using the modular properties of the Jacobian functions [6].

3.2. The spin-1/2 dynamics

Using the above described classical solutions for the charge motion, the laser-spin interaction (6) reduces to the $\mathrm{H}(t) := -\boldsymbol{\Omega}(t) \cdot \mathbf{S}$, with the following Larmor vector $\boldsymbol{\Omega}(t)$

$$\Omega_1(t) = \frac{1}{2}\eta\omega_L'\sqrt{1-\varepsilon^2}[(g+1)\mathrm{dn}(\omega_L't\,\mu)-\gamma_z]\mathrm{cn}(\omega_L't,\mu), \tag{12}$$

$$\Omega_2(t) = \frac{1}{2}\omega_L'\varepsilon\left[(g+1)\mathrm{dn}(\omega_L't,\mu)-\gamma_z(1-\mu^2)\right]\mathrm{sn}(\omega_L't\,\mu), \tag{13}$$

$$\Omega_3(t) = -\frac{1}{2}\eta^2\omega_L\varepsilon\sqrt{1-\varepsilon^2}\left[g-\gamma_z\,\mathrm{dn}(\omega_L't,\mu)\right]. \tag{14}$$

So far, a particle's spin-1/2 effectively interacts with a laser via the spatially homogeneous, time depending magnetic field given by a superposition of the alternating magnetic field (12)-(13) in a plane orthogonal to the wave propagation and a small, "almost constant" magnitude magnetic field (14) along the laser beam. Note that, in the leading order expansion over the powers of modulus μ expressions (12)-(14) reduce to the non-relativistic Larmor vector corresponding to the laser magnetic field.

4. Non-dipole effects examples

In this section, based on the equation (5), we make an outline of the derivation of two non-dipole effects in spin evolution due to the interaction particle interaction with a strong elliptically polarized laser: non-linear dependence of spin-precession frequency on the laser frequency, and the resonance character of spin-flip transitions.

4.1. Spin effects under the cyclic evolution

Consider at first a spin evolution in the linearly polarized laser field. In this case according to expressions (12)-(14) the spin-1/2 wave function displays non-trivial periodicity:

$$|\chi(t+4\mathrm{K}(\mu)/\omega')\rangle = |\chi(t)\rangle, \tag{15}$$

where $\mathrm{K}(\mu)$ is the quarter period of the Jacobian function. Apart from the pure kinematic non-relativistic Doppler shift, the particle's spin precession frequency depends non-linearly on the laser intensity and polarization through the period $\mathbb{K}$. For the semi-relativistic intensities $\eta \ll 1$, the period of the particle oscillation can be represented in the form of the expansion

$$T_P = \frac{2\pi}{\omega'}\left[1+\left(\frac{1}{2}\right)^2 1-2\varepsilon^2\gamma_z^2\eta^2+\left(\frac{1\cdot 3}{2\cdot 4}\right)^2\frac{(1-2\varepsilon^2)^2}{(1-\beta_z)^4}\eta^4+\dots\right].$$

The presence of $\mathbb{K}$ in (15) exposes a fundamental peculiarity of the particle dynamics which is beyond the dipole approximation.

4.2. The spin flip resonance in a laser field

Our computations show, that in a strong laser instead of the well-known dipole approximation result, vanishing spin-flip amplitude, the resonance transition occurs for certain values of the laser field strength, η_*^2. The position of the resonance point η_*^2 depends on particle's gyromagnetic ratio and the laser field polarization.

4.2.1. Circularly polarized laser Let us at first consider motion in the circularly polarized background. In this case an explicit form of the resonance can be derived [9]. In this case spin evolution problem reduces to the well-known Rabi problem [10] and the spin-flip probability reads

$$\mathcal{P}_{\downarrow\uparrow} = \frac{\kappa^2\eta^2}{\kappa^2\eta^2 + (\eta^2 - \eta_*^2)^2}\, \sin^2(\omega_S t)\,. \tag{16}$$

An intensity resonance occurs at $\eta_*^2 := 4/(g-1)$ and the spin probability oscillates with the frequency ω_S

$$\omega_S := \frac{\omega_L|1-g|}{8}\,\sqrt{\kappa^2\eta^2 + (\eta^2 - \eta_*^2)^2}\,, \tag{17}$$

determined not only by ω_L, but the laser intensity as well. In (16) the constant $\kappa^2 := 2\,g^2/(1-g)^2$ was introduced.

4.2.2. Elliptically polarized laser In order to study the generic case of elliptically polarised laser we adopt the method of reducing the system of linear equations (5) to the analysis of the Riccati equation (see e.g. [11]).

The evolution operator $U(t, t_0)$ for (5) can be represented in the factor form using the operators $S_\pm = 1/2\,(\sigma_1 \pm \imath\,\sigma_2)$ and $S_0 = 1/2\,\sigma_3$:

$$U(t,0) = \exp\left(a(t)S_+\right)\exp\left(b(t)S_0\right)\exp\left(c(t)S_-\right)\,. \tag{18}$$

Three unknown functions $a(t), b(t)$ and $c(t)$ determine a generic spin-1/2 state. Noting that the probabilities of transitions between states labelled by spin-1/2 projection to z-axis are given by formulas

$$\mathcal{P}_{\uparrow\uparrow} = \frac{1}{1+|a|^2}\,, \qquad \mathcal{P}_{\uparrow\downarrow} = \frac{|a|^2}{1+|a|^2}\,, \tag{19}$$

we will concentrate on the determination of function $a(t)$. From the spin evolution equation for (18) it follows that $a(t)$ is subject to the Riccati equation:

$$\dot{a} = \imath(\Omega_+ a^2 + \Omega_0 a - \Omega_-)\,. \tag{20}$$

with functions $\Omega_\pm = 1/2\,(\Omega_1 \pm \imath\,\Omega_2)$ and $\Omega_0 = 1/2\,\Omega_3$.

Our numerical studies of (20) allows to determine transition probabilities (19) for different values of radiation intensities, polarization and the particle's gyromagnetic ratio. From the curve depicted on Figure 1 one can see the dependence of spin-flip transition from a laser intensity for for fixed value of gyromagnetic ratio (g=6) and light polarization. Analysis shows that the value of intensity resonance point η_*^2 non-trivially depends on the values a laser polarization. As an example, the Figure 2 shows the *"resonance intensity curves"*, i.e., points on the $(\eta, g)-$plane for which the spin-flip occurs with the unit probability for two polarization, the circular one and elliptical with $\varepsilon = 1/2$.

5. Conclusion
In conclusion, our studies point on few spin effects associated with a spin-1/2 charged particle motion in laser fields whose intensities provide the semi-relativistic interaction regime. These effects are identified beyond the dipole approximation and could be attributed to the fact that the magnetic forces have been taken into account and alter the spin evolution appreciably.

Finally, it is worth to point out some flaws of our study. First of all, the results stated above are strongly based on the semiclassical approach used. Therefore it is necessary to convince
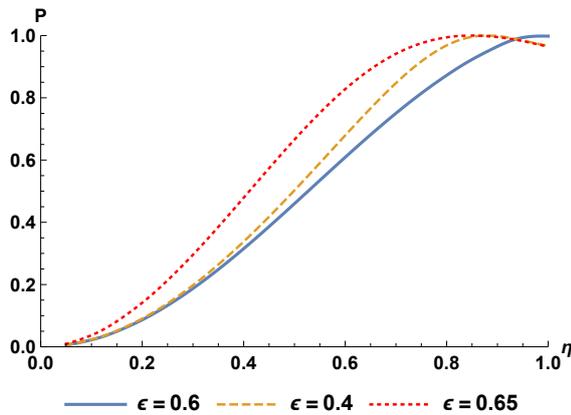
Figure 1. Spin-flip probability $\mathcal{P}$ vs. intensity η, gyromagnetic ratio g=6.
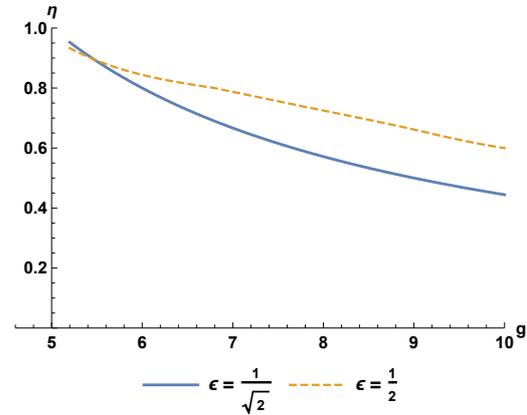
Figure 2. Typical resonance curves on (η, g)-plane for different polarisation.

that the resonant character of spin flip is not a spurious feature due to a fault approximation. A complete relativistic consideration within the Dirac theory or using the Bargmann, Michel & Telegdi spin evolution equations [12] is required. Besides this, it should be noted that the effects of a back reaction of a radiation as well spin forces distorting the spinning particle's trajectory (cf. e.g. the discussion in [13],[14]), [15] may alert our conclusions.

Acknowledgments

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