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ELEMENTARY PARTICLES AND FIELDS
Theory

On Precession of Entangled Spins in a Strong Laser Field*

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Abstract—A dynamics of the entanglement under an environmental influence is modelled by a bound state composed of two heavy particles interacting with a strong laser. Adopting the semiclassical attitude, a trajectory of the bound state's center-of-mass is found from the Newton equations solved beyond the dipole approximation and taking into account the magnetic field effect. At the same time the dynamics of constituent spins under the laser coupling is studied quantum mechanically solving the nonrelativistic von Neumann equation with the effective Hamiltonian determined by the bound state's classical trajectory. Based on the solution, the effects of an intense linearly polarized monochromatic plane wave on the precession of entangled spins are discussed for a specific kind of mixed initial states including a family of maximally entangled Werner states.

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1. INTRODUCTION

This article addresses the question how a high-intensity laser interacting with a charged composite system affects the evolution of its entangled subsystems (for the definition and properties of the entanglement see, e.g., recent books [1–3]). To understand the dynamics of entanglement under a laser coupling a simple model is formulated. A laser beam is modelled by a strong linearly polarized monochromatic electromagnetic plane wave and a composite system is represented by a bound state consisting of two heavy spin-1/2 particles.

The behavior of a nonrelativistic charged particle driven by a low intensity laser is completely determined by the electric component of electromagnetic field with no mention at all of its magnetic component. The electric-field dominance together with the dipole approximation provides a consistent solution to the equation of motion for a particle classical trajectory and allows to determine the dynamics of spin degrees of freedom [4]. However, as an intensity of radiation is growing up, an accelerating-particle velocity can attain the relativistic values [5, 6]. As a result the dipole approximation becomes inconsistent and the magnetic part of the Heaviside–Lorentz force is not negligible any more. This, in turn, makes an influence on a particle spin evolution unavoidable.

In the present article, based on the recent results [7], we make a step forward to the relativistic description and consider a particle motion beyond the dipole approximation and taking into account the magnetic-field effect. We neglect the influence of a particle spin on the classical orbit and consider the spin evolution quantum mechanically as precession in a certain spatially homogeneous magnetic field configuration which is determined solely by a particle classical trajectory[3)]. Having derived the effective Hamiltonian for spin degrees we analyze the influence of a laser intensity on dynamics of the entanglement.

In accordance with the above drawn program we formulate at first the model describing the interaction of a charged composite system with an intense monochromatic plane wave radiation. Then, in Section 2, the semiclassical evolution of the model is discussed. Section 3 is devoted to a description of the classical trajectory of a composite system center-of-mass as function of a laboratory-frame time. Besides the classical treatment the results on quantum dynamics of spin degrees of freedom in the linearly polarized laser field are stated as well. In the next section the detailed derivation of the oscillation of entanglement is given for two special initial spin configurations including the so-called Werner states [3]. For the Werner state the condition of the entanglement stability under a laser coupling is formulated. Finally,

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[3)]The discussion of such an approximation can be found, e.g., in [8].

in Section 5, a few concluding remarks are given. Appendices A and B contain some technical formulas used in the main text.

2. THE MODEL FORMULATION

The model under consideration is formulated as follows. A bound state with charge $-q_B$ interacting with a laser radiation is modeled by the monochromatic plane wave propagating along the z-axis

$$\mathbf{A}(t,\mathbf{x}) := a\left(\varepsilon\cos(\omega_L\xi),\ \sqrt{1-\varepsilon^2}\sin(\omega_L\xi),\ 0\right), \quad \xi = t - \frac{z}{c}. \tag{1}$$

In (1) the parameter $\varepsilon \in [0,1]$, is the light polarization parameter, ω_L is the wave frequency and the constant a measures the intensity of a radiation. The constant a sets the scale for the dimensionless parameter η, the so-called laser field strength [9, 10]:

$$\eta^2 = \frac{q_B^2 a^2}{M_B^2 c^4}.$$

Here, M_B is the mass of a bound state traveling in the electromagnetic background (1). The bound state is considered to be composed of two heavy particles (p,n) with the binding potential

$$V_B = V_0(r) + V_{SS}, \quad V_{SS} := V_S(r)\mathbf{s}^{(n)} \otimes \mathbf{s}^{(p)}, \tag{2}$$

where $V_0(r)$ and $V_S(r)$ are scalar functions of the relative distance between bound state's constituents, $r = |\mathbf{r}_n - \mathbf{r}_n|$, the vectors $\mathbf{s}^{(n)}$ and $\mathbf{s}^{(p)}$ denote the spin of constituent (n) and constituent (p), respectively.

Concerning the coupling of a bound state with an external radiation the mixed type of interaction is proposed; a direct bound-state charge–laser interaction—V_{CL}, and spin–laser coupling—V_{SL} via the individual magnetic moments of constituents. The former charge–laser term V_{CL} is chosen supposing that the bound state interacts as a whole with radiation via a point electric charge q_B positioned at the center of mass $\mathbf{R} = \frac{m^{(n)}\mathbf{r}_n + m^{(p)}\mathbf{r}_p}{m^{(n)}+m^{(p)}}$,

$$V_{\mathrm{CL}} := \frac{q_B}{c}\mathbf{v}_R \cdot \mathbf{A}(t,\mathbf{R}), \quad \mathbf{v}_R := \frac{d\mathbf{R}}{dt}. \tag{3}$$

Apart from the effective charge–laser interaction (3), we take into account the coupling of laser field to each constituent spin in the relativistically modified Larmor form:

$$V_{\mathrm{SL}} := -\boldsymbol{\Omega}^{(n)}(t,\mathbf{r}_n)\cdot\mathbf{s}^{(n)} - \boldsymbol{\Omega}^{(p)}(t,\mathbf{r}_p)\cdot\mathbf{s}^{(p)}, \tag{4}$$

where vector $\boldsymbol{\Omega}^{(i)}$ reads

$$\boldsymbol{\Omega}^{(i)} := \frac{e^{(i)}g^{(i)}}{2m_{(i)}c}\left(\mathbf{B} - \frac{1}{c}\left[\mathbf{v}^{(i)}\times\mathbf{E}\right]\right) + \frac{1}{2c^2}\left[\mathbf{v}^{(i)}\times\mathbf{a}^{(i)}\right]. \tag{5}$$

Here, vectors $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic components of a laser field evaluating along the trajectory of the ith particle (with charge $e^{(i)}$, mass $m_{(i)}$, and gyromagnetic ratio $g^{(i)}$) moving in the laboratory frame with the velocity $\mathbf{v}^{(i)}$ and acceleration $\mathbf{a}^{(i)}$. This form of spin–laser coupling has a clear meaning. The first term in parentheses is the magnetic field in the instantaneous rest frame of a charged particle, while the last contribution in (5) is the so-called leading Thomas-precession correction [11] due to the nonvanishing curvature of a particle trajectory, cf., e.g., [4].

Gathering all the above together, the evolution of a bound state travelling in laser background is governed by the total Hamiltonian

$$H = H_0 + V_{SS} + V_{\mathrm{CL}} + V_{\mathrm{SL}}, \tag{6}$$

where H_0 is the Hamiltonian of free spinless constituents. Note that, in accordance with the semiclassical picture we follow, the contribution to the phase of our system's wave function coming from the spin–laser interaction term, V_{SL}, is negligibly small in the leading approximation. This term in (6) will come into play only later when we turn to a study of dynamics of spin degrees.

Concluding the model formulation it is worth to stress that the consideration given below is well justified only if the bound state constituents move with the nonrelativistic relative velocities. Mean time, concerning the velocity of the center of mass, the used approach is still valuable for the semirelativistic regime, $\boldsymbol{v}_R/c \sim 1$, which may be attainable due to the acceleration of a bound state as a whole by a laser field.

3. THE CLASSICAL AND QUANTUM DYNAMICS

In this section the semiclassical evolution of our model is described. At first the classical dynamics of the center of mass of a bound state will be presented. Afterwards, based on these results, the quantum evolution of the spins of constituents will be established.

3.1. Classical Trajectory for a Bound-State Center of Mass

Ignoring for a moment radiation coupling to the spins of constituents consider the dynamics of a bound state governed by the Hamiltonian $H_0 + V_{SS} + V_{\mathrm{CL}}$. Since this Hamiltonian admits separation of the relative and absolute motion, we are able to write down the trajectory of a bound-state center of mass exploiting the exact solution to the analogous Hamilton–Jacobi problem for a single charged particle found in [7].

Based on the solution given in [7] one can verify that the bound-state center of mass moves along the trajectory $\mathbf{R}(t)$ given by the following expressions[4)]:

$$R_x(t) = -\frac{c}{\omega_L}\sqrt{\frac{\varepsilon^2}{1-2\varepsilon^2}}\arcsin\left[\mu\mathrm{sn}(\omega_L' t,\mu)\right], \quad (7)$$

$$R_y(t) = \frac{c}{\omega_L}\sqrt{\frac{1-\varepsilon^2}{1-2\varepsilon^2}} \quad (8)$$
$$\times \ln\left[\frac{\mu\mathrm{cn}(\omega_L' t,\mu)+\mathrm{dn}(\omega_L' t,\mu)}{1+\mu}\right],$$

and

$$R_z(t) = ct - \frac{c}{\omega_L}\mathrm{am}(\omega_L' t,\mu). \quad (9)$$

The trajectory (7)–(9) is expressed in terms of the Jacobian elliptic functions $\mathrm{sn}(z,\mu)$, $\mathrm{cn}(z,\mu)$, $\mathrm{dn}(z,\mu)$ and amplitude function $\mathrm{am}(z,\mu)$ [12] whose modulus is[5)]:

$$\mu^2 = \eta^2\frac{1-2\varepsilon^2}{(1-\beta_z)^2}.$$

The argument of Jacobian functions represents the laboratory-frame time, t, scaled by the nonrelativistically-Doppler-shifted laser frequency $\omega_L' = (1-\beta_z)\omega_L$, $\beta_z = (\mathbf{v}_R)_z(0)/c$. From this solution it follows that the velocity of the center of mass is

$$\mathbf{v}_R = \left(-c\eta\varepsilon\mathrm{cn}\left(\omega_L' t,\mu\right), \quad (10)\right.$$
$$\left.-c\eta\sqrt{1-\varepsilon^2}\mathrm{sn}\left(\omega_L' t,\mu\right), c - c(1-\beta_z)\mathrm{dn}\left(\omega_L' t,\mu\right)\right).$$

The Jacobian functions in (10) tell us that components of a charged-particle velocity in the plane orthogonal to the wave propagation are periodic functions of time with the period[6)]

$$T_P = \frac{4\mathrm{K}}{\omega_L'} := \frac{2\pi}{\omega_P}, \quad (11)$$

while in the direction of propagation the oscillation period is twice smaller ($T_P/2$). The fundamental circular frequency of the particle motion, ω_P, differs from the frequency ω_L of a laser field due to the nonlinearities of the dynamical equations taken into account.

3.2. The Evolution of Spin Degrees

Now we pass to a second issue of our study, below the evolution of the spin degrees is discussed. The classical orbit described in the preceding section will be used to evaluate forces acting on the spins of constituents, while a bound state is moving in a laser background.

We follow the conventional quantum mechanical approach when a spin-j state is described by the $(2j+1)\times(2j+1)$ density matrix ϱ, that evolves according to the nonrelativistic von Neumann equation

$$\dot{\varrho}(t) = -\frac{i}{\hbar}[H_S(t),\varrho(t)]. \quad (12)$$

For the problem under consideration the effective spin Hamiltonian H_S is defined as the Hamiltonian $V_{SS}+V_{\mathrm{SL}}$ projected to the constituent-particle classical trajectory:

$$H_S(t) \quad (13)$$
$$= \{V_{SS}+V_{\mathrm{SL}}\}\Big|_{\text{Classical trajectory}}.$$

Evaluating (13), two approximations, consistent with our assumption on the large mass of the bound state constituents are made. Namely, we “freeze” the relative motion of constituents inside the bound state, i.e., approximate their relative trajectory by its mean value $r(t)=\overline{r}$ and, correspondingly, neglect all contributions of order υ_r/c, where υ_r is the relative velocity of constituents. Within this type of the Born–Oppenheimer approximation [13] the effective spin–laser Hamiltonian (13) admits the following decomposition:

$$H_S = -\mathfrak{B}^{(n)}(t)\cdot\mathbf{s}^{(n)}\otimes I \quad (14)$$
$$- I\otimes\mathbf{s}^{(p)}\cdot\mathfrak{B}^{(p)}(t) + H_I.$$

Our calculations using expressions (7)–(9) for a trajectory of the bound-state center of mass, show that the effective-time-depending potential $\mathfrak{B}^{(i)}(t)$, $i=(n,p)$, reads

$$\mathfrak{B}_x^{(i)}(t) = \eta\frac{\omega_L'}{2}\sqrt{1-\varepsilon^2} \quad (15)$$
$$\times\left[(\widetilde{\mathrm{g}}^{(i)}+1)\mathrm{dn}(\omega_L' t,\mu) - (1-\beta_z)\right]\mathrm{cn}(\omega_L' t,\mu),$$
$$\mathfrak{B}_y^{(i)}(t) = \eta\frac{\omega_L'}{2}\varepsilon\Big[(\widetilde{\mathrm{g}}^{(i)}+1)\mathrm{dn}(\omega_L' t,\mu)$$

[4)]The condition $\mathbf{R}(0)=0$ is supposed and the frame is fixed, where time average of a bound-state velocity component, orthogonal to the wave propagation, is vanishing: $\langle\langle\mathbf{v}_R^{\perp}\rangle\rangle = 0$.

[5)]It is assumed that μ is from the fundamental domain, $0<\mu^2<1$. The solution outside the fundamental domain follows from the modular properties of the Jacobian functions, cf. details in [7].

[6)]Note, that the quarter period K of the Jacobian elliptic function depends nonlinearly on laser intensity and its polarization.

$$-(1-\beta_z)(1-\mu^2)\Big]\mathrm{sn}(\omega'_L t,\mu),$$

$$\mathfrak{B}_z^{(i)}(t) = -\eta^2\frac{\omega'_L}{2}\varepsilon\sqrt{1-\varepsilon^2}\left[\frac{\widetilde{\mathrm{g}}^{(i)}}{1-\beta_z} - \mathrm{dn}(\omega'_L t,\mu)\right],$$

where $\widetilde{\mathrm{g}}^{(i)} = \left(e^{(i)}/m_{(i)}\right)(M_B/q_B)\,\mathrm{g}^{(i)}$.

The spin–spin interaction term H_I in (14) originates from the potential V_{SS} under the same static approximation for the spatial relative degrees of freedom:

$$\hbar H_I = g\mathbf{s}^{(n)}\otimes\mathbf{s}^{(p)}. \qquad (16)$$

The constant g in (16) is determined by the spin–spin potential evaluated at the mean value of the relative distance between constituents, $g := \hbar V_S(\overline{r})$.

3.2.1. Single spin-1/2 dynamics in the linearly polarized laser field. Now we fix the spin of constituents, $\mathbf{s}^{(i)} = \hbar\boldsymbol{\sigma}/2$[7], and restrict ourselves by considering the linearly polarized radiation (1) with $\varepsilon = 0$. Before studying a two-spin system let at first write down the evolution operator for a single, say $\mathbf{s}^{(n)}$-spin precession problem with an external magnetic field of the form (15).

For the linearly-polarized case the effective magnetic field (15) significantly simplifies and we arrive at the exactly solvable problem; spin-1/2 precession in time depending magnetic field directed along the x-axis. The solution for the density matrix with the initial condition, $\varrho(0) = \varrho_0$, reads

$$\varrho(t) = U_{(n)}(t)\varrho_0 U^+_{(n)}(t), \qquad (17)$$

where

$$U_{(n)}(t) = \exp\left(\frac{i}{2}\vartheta^{(n)}(t)\sigma_1\right). \qquad (18)$$

The explicit form of the phase factor $\vartheta^{(n)}(t)$ can be easily calculated:

$$\vartheta^{(n)}(t) := \int\limits_0^t d\tau\mathfrak{B}_x\Big|_{\varepsilon=0} \qquad (19)$$

$$= \frac{\eta}{2}\left[\widetilde{\mathrm{g}}^{(n)}+1\right]\mathrm{sn}(\omega'_L t,\mu) - \frac{1}{2}\arcsin\left[\mu\,\mathrm{sn}(\omega'_L t,\mu)\right].$$

The angle (19) contains nontrivial dependence on laser intensity via the particle fundamental circular frequency ω_P, cf. (11). Only for small intensities $\eta \ll 1$ the expression for the angle ϑ reduces to the well-known nonrelativistic precession result:

$$\vartheta_{\mathrm{NR}} := \frac{1}{2}\eta\widetilde{\mathrm{g}}^{(n)}\sin(\omega_L t). \qquad (20)$$

[7] The spin-1/2 is assumed to be in a generic mixed state with the standard density matrix parametrization: $\varrho = \frac{1}{2}(I + \chi\mathbf{u}\cdot\boldsymbol{\sigma})$, $\mathbf{u}^2 = 1$, $0 < \chi < 1$.

3.2.2. Interacting spins in the linearly polarized laser field. Having results of the preceding paragraphs one can study the evolution of constituent's spins taking into account the spin–spin interaction term (16) as well.

We start with the observation, that the tensor product of the single spin evolution operators

$$W(t) := U_{(n)}(t)\otimes U_{(p)}(t),$$

"gauges" out the Hamiltonians of both subsystems, while under the action of the operator W the interaction Hamiltonian (16) changes as (see Appendix B for notations and useful formulas):

$$H'_I(t) := W^+ H_I W \qquad (21)$$
$$= g\hbar\left(\cos\vartheta_-\boldsymbol{\sigma}\otimes\boldsymbol{\sigma} + \sin\vartheta_-\sigma_{[12]}\right),$$

where

$$\vartheta_-(t) := \vartheta^{(n)} - \vartheta^{(p)} = \frac{1}{2}\eta\left(\widetilde{\mathrm{g}}^{(n)} - \widetilde{\mathrm{g}}^{(p)}\right)\mathrm{sn}(\omega'_L t,\mu).$$

Therefore, the evolution operator for the interacting spins is convenient to represent as

$$U(t) = W(t)X(t). \qquad (22)$$

The unknown operator X is subject to the equation

$$\dot{X}(t) = -\frac{i}{\hbar}H'_I(t)X(t). \qquad (23)$$

This equation admits the formal solution with T-exponent factor:

$$X(t) = e^{ig\psi(t)\boldsymbol{\sigma}\otimes\boldsymbol{\sigma}}\,\mathrm{T}\left(\exp\frac{i}{\hbar}\int\limits_0^t V_I(\tau)d\tau\right), \qquad (24)$$

where

$$\psi(t) := \int\limits_0^t dt\cos\vartheta_-(t), \qquad (25)$$

and

$$V_I(t) := g\hbar \qquad (26)$$
$$\times\sin\vartheta_-\left[\cos(4g\psi(t))\sigma_{[12]} + \frac{3}{4}\sin(4g\psi(t))\sigma_{[30]}\right].$$

The first factor in (24) has a remarkable Eulerian exponent-type representation[8]:

$$e^{ig\psi(t)\boldsymbol{\sigma}\otimes\boldsymbol{\sigma}} = \frac{1}{2}e^{ig\psi(t)} \qquad (27)$$
$$+\frac{1}{2}e^{-ig\psi(t)}\left[\cos(2g\psi(t)) + i\boldsymbol{\sigma}\otimes\boldsymbol{\sigma}\sin(2g\psi(t))\right],$$

[8] For completeness the derivation of the Euler-type representation for the exponent from the scalar tensorial product of sigma matrices is given in Appendix B.

while the T-exponent factor in (24) is much cumbersome one. However, it turns into unity if the constituents have equal gyromagnetic ratios. Note also that in the leading order, for small laser intensities, the interaction potential V_I reads

$$V_I(t) = \frac{\hbar}{2} g\eta \left(\widetilde{\mathrm{g}}^{(n)} - \widetilde{\mathrm{g}}^{(p)}\right) \sin(\omega_L t) \qquad (28)$$
$$\times \left[\cos(4gt)\sigma_{[12]} + \frac{3}{4}\sin(4gt)\sigma_{[30]}\right] + O(\eta^2).$$

4. EVOLUTION OF THE ENTANGLED STATES

We arrived now at a place where the intrigued quantal phenomenon, *entanglement*, comes into play. Analyzing this phenomenon, it is very important to understand the dynamics of entanglement under the environment coupling [14]. In our model the environment is realized as a background laser radiation. In this context, below the effect of a laser coupling on the quirk of fate of the entangled state is analyzed.

The most generic density matrix for two spins written in the so-called Fano form [3],

$$\varrho_{2\times 2} = \frac{1}{4}\left(I \otimes I + \alpha_i \sigma_{0i} + \beta_i \sigma_{i0} + \gamma_{ij}\sigma_{ij}\right), \quad (29)$$

is characterized by a set of 15 parameters α_i, β_i, and γ_{ij}, $i, j = 1, 2, 3$. The density matrix is non-negative matrix and the requirement of non-negativity imposes the following set of algebraic inequalities on these parameters:

$$\sum_{\text{2-principal minors}} \mathrm{tr}_2 \varrho_{2\times 2} \geq 0, \qquad (30)$$
$$\sum_{\text{3-principal minors}} \mathrm{tr}_3 \varrho_{2\times 2} \geq 0, \quad \det \varrho_{2\times 2} \geq 0.$$

We postpone for the future analysis the evolution of a generic density matrix (29) and deal here only with the initial density matrices $\varrho_{2\times 2}(0)$ of two special types.

Werner states. Consider at first a family of entangled mixed states[9)], the so-called Werner states [15], characterized by a single real parameter,

$$\varrho_{\mathrm{W}} \qquad (31)$$
$$:= \begin{pmatrix} (1+p)/4 & 0 & 0 & p/2 \\ 0 & (1-p)/4 & 0 & 0 \\ 0 & 0 & (1-p)/4 & 0 \\ p/2 & 0 & 0 & (1+p)/4 \end{pmatrix}.$$

[9)] It is worth to mention that Werner states enjoy the largest entanglement accessible by unitary transformations [16].

In compact notations the Werner state (32) can be written as

$$\varrho_{\mathrm{W}} = \frac{1}{4}\left(I \otimes I + p\boldsymbol{\sigma} \otimes \boldsymbol{\sigma}\right). \qquad (32)$$

The parameter p in (32) is simply related to the important characteristics of quantum states, the so-called *fidelity* (F), which measures the overlap of a given Werner state with the maximally entangled pure Bell state:

$$p = \frac{4F - 1}{3}.$$

According to (30), the matrix (32) is non-negative for values of parameter p from the closed interval

$$-1/3 \leq p \leq 1. \qquad (33)$$

For the fidelity $F \leq 1/2$ the Werner state is unentangled, while if

$$1/3 < p \leq 1, \qquad (34)$$

Werner density matrix describes the mixed entangled state.

To find the fate of the entanglement of the initial Werner state note that since the entanglement properties are invariant under the local unitary transformation [2] of the form $W(t) := U_{(n)}(t) \otimes U_{(p)}(t)$, only the action of the operator $X(t)$ may change the entanglement. Moreover, in that action only the T-exponent factor is a relavant one. With this observation one can easily evaluate the leading, in laser intensity, change of the density matrix

$$\delta_t \varrho_{\mathrm{W}} = \frac{i}{\hbar}[V_I, \varrho_{\mathrm{W}}] = -\frac{1}{2} g\eta p \left(\widetilde{\mathrm{g}}^{(n)} - \widetilde{\mathrm{g}}^{(p)}\right) \qquad (35)$$
$$\times \sin(\omega_L t)\left[\cos(4gt)\sigma_{[30]} + \frac{3}{4}\sin(4gt)\sigma_{[12]}\right].$$

Introducing the partial traced matrices

$$\varrho_n(t) := \mathrm{Tr}_p U(t) \varrho_{\mathrm{W}} U^+(t), \qquad (36)$$
$$\varrho_p(t) := \mathrm{Tr}_n U(t) \varrho_{\mathrm{W}} U^+(t),$$

the subsystem properties can be analyzed. Particularly, using (35), it follows that the maximally mixed initial state, $\varrho_n(0) = \frac{1}{2}I$ evolves to a state corresponding to a spin whose projection along the z-axis is oscillating according to the equation:

$$\langle s_z^{(n)} \rangle = \hbar\eta p \left(\widetilde{\mathrm{g}}^{(n)} - \widetilde{\mathrm{g}}^{(p)}\right) \qquad (37)$$
$$\times g\left[\frac{\sin^2(\omega_L/2 + 2g)t}{\omega_L + 4g} + \frac{\sin^2(\omega_L/2 - 2g)t}{\omega_L - 4g}\right].$$

Note, deriving (37) it was assumed that $\omega_L^2 \neq 16g^2$.

The "$\alpha\beta\gamma$" state. Let us consider another special three-parameter family of density matrices given by

$$\varrho_0 = \frac{1}{4}\Bigg(\mathrm{I} + \alpha\frac{1}{2}\left(\sigma_{03}+\sigma_{30}\right) \qquad (38)$$
$$+ \beta\frac{1}{2}\left(\sigma_{03}-\sigma_{30}\right) + \frac{1}{2}\gamma\left(\sigma_{12}-\sigma_{21}\right)\Bigg).$$

One can verify that ϱ_0 has the eigenvalues:

$$\lambda_1 = \frac{1}{4}(1-\alpha), \quad \lambda_2 = \frac{1}{4}(1-\sqrt{\beta^2+\gamma^2}), \qquad (39)$$
$$\lambda_3 = \frac{1}{4}(1+\sqrt{\beta^2+\gamma^2}), \quad \lambda_4 = \frac{1}{4}(1+\alpha).$$

Therefore, the non-negativity of the density matrix is provided by α, β, and γ if and only if:

$$\alpha^2 \le 1, \quad \beta^2+\gamma^2 \le 1. \qquad (40)$$

Having in mind this restriction to the moduli parameters consider the evolution of the partial traced matrices (36). We state results only for the reduced density matrix ϱ_n, the corresponding expressions for the spin $\mathbf{s}^{(p)}$ can be derived in a similar way. With the aid of formulas collected in the Appendix A we obtain

$$\varrho_n(t) := \frac{1}{2}\left(I + \mathbf{a}\cdot\boldsymbol{\sigma}\right), \qquad (41)$$

where $\mathbf{a}$ stands for the vector

$$\mathbf{a} = \frac{1}{2} \qquad (42)$$
$$\times\,(0, \sin\vartheta_+\cos\vartheta_- F_n(t) - \cos\vartheta_+\sin\vartheta_- G_n,$$
$$\cos\vartheta_+\cos\vartheta_- F_n(t) - \sin\vartheta_+\sin\vartheta_- G_n).$$

In (42) two angles $\vartheta_\pm := \vartheta^{(n)} \pm \vartheta^{(p)}$ and functions:

$$F_n(t) := \frac{1}{2}\left(\alpha - \beta\cos(4gt) + \frac{3}{4}\gamma\sin(4gt)\right), \quad (43)$$

$$G_n(t) := \frac{1}{2}\left(\alpha\cos(4gt) - \beta\right) \qquad (44)$$

were introduced. As (42) shows, $\mathbf{s}^{(n)}$ oscillates in a very complicated manner. The amplitude and the frequency of the oscillation depend on the coupling between spins as well as laser field intensity. The detailed analysis of the spin precession will be given in forthcoming publications. Below, only a few transparent results on spin oscillation will be given.

The modulus $|\mathbf{a}|$ measures the deviation from a pure state, if $|\mathbf{a}| < 1$ the spin is in mixed state. According to (42)

$$\mathbf{a}^2 = (\cos\vartheta_- F_n + \sin\vartheta_- G_n)^2 \qquad (45)$$
$$- \sin(2\vartheta_-)(1+\cos(2\vartheta_+))F_n G_n.$$

If the constituents have equal magnetic moments, formula (45) reduces to the expression depending only on the coupling constant between spins:

$$\mathbf{a}^2 = F_n^2. \qquad (46)$$

For nonzero $\widetilde{\mathrm{g}}^{(n)} - \widetilde{\mathrm{g}}^{(p)}$, the spin evolution depends nonlinearly on a laser intensity. In the leading, $O(\eta^2)$, order the modulus $|\mathbf{a}|$ oscillates as

$$\mathbf{a}^2 = F_n^2(t) + \frac{1}{4}\eta^2(\widetilde{\mathrm{g}}^{(n)} - \widetilde{\mathrm{g}}^{(p)})^2\sin^2(\omega_L t)G_n^2(t). \qquad (47)$$

If the initial state is chosen with $\alpha = -\beta$ and $\gamma = 0$ (both spins along the z-axis) Eq. (47) simplifies to

$$\mathbf{a}^2 = \alpha^2\cos^2(2gt) \qquad (48)$$
$$\times\left[1 + \frac{1}{4}\eta^2(\widetilde{\mathrm{g}}^{(n)} - \widetilde{\mathrm{g}}^{(p)})^2\sin^2(\omega_L t)\right].$$

This tells that the spin of an individual constituent oscillates between the initial state and maximally mixed state which is attainable at moments $t = (2n+1)\pi/(4g)$, $n \in \mathbb{Z}$.

5. CONCLUDING REMARKS

In the present note we aimed to study some features of a strong laser effect on a composite system which are sensitive to the intensity of the electromagnetic radiation. We formulated the model for a laser interaction with the bound state composed from two heavy particles. The relative motion of constitutes was treated in the spirit of the Born–Oppenheimer method [13] and the semiclassical approximation has been used to find the evolution operator. According to our result, if the bound-state constituents have equal magnetic moments, then the entanglement properties of the initial spin configuration chosen as the Werner state are unchangeable under the evolution. For the constituents with different gyromagnetic ratios the entanglement of our model evolves with time in a very complicated manner, depending on the intensity of the laser beam as well as on the coupling between spins.

Two generic problems—quantification of the degree of entanglement and classification of nonlocalities exposing by a composite quantum-mechanical system, are of fundamental importance in the quantum information theory as well as the quantum mechanics itself. In the proposed model we plan to investigate both issues. Particularly, we intend to perform the detailed analysis of various characteristics of the entanglement, including the evolution of the concurrence for pure and mixed initial states in a strong laser environment.

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APPENDIX

6. THE FANO BASIS AND A LITTLE BIT ALGEBRA

For some applications an arbitrary 4×4 Hermitian matrice is convenient to represent in the so-called Fano basis, written as

$$I \otimes I, \quad \sigma_{0j} := I \otimes \sigma_j, \quad \sigma_{j0} := \sigma_j \otimes I, \tag{A.1}$$
$$\sigma_{ij} := \sigma_i \otimes \sigma_j, \quad i, j = 1, 2, 3,$$

or in dense notation simply as

$$\sigma_{\mu\nu} := \sigma_\mu \otimes \sigma_\nu, \quad \sigma_\mu := (I, \sigma_i), \quad \mu, \nu = 0, 1, 2, 3.$$

Here, σ_1, σ_2, and σ_3, are the Pauli $\boldsymbol{\sigma}$ matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tag{A.2}$$
$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

satisfying the algebra

$$[\sigma_i, \sigma_j] = 2i\varepsilon_{ijk}\sigma_k. \tag{A.3}$$

In the main text and in formulae below the following notation is used also

$$\sigma_{[\mu\nu]} := \sigma_\mu \otimes \sigma_\nu - \sigma_\nu \otimes \sigma_\mu,$$
$$\sigma_{\{\mu\nu\}} := \sigma_\mu \otimes \sigma_\nu + \sigma_\mu \otimes \sigma_\nu.$$

Elements of the Fano basis obey the following commutator, anticommutator relations with the scalar tensorial product of the Pauli matrices, $M := \boldsymbol{\sigma} \otimes \boldsymbol{\sigma}$,

$$[\sigma_{ij}, M] = 2i\varepsilon_{ijk}\sigma_{[k0]}, \tag{A.4}$$
$$\{\sigma_{ij}, M\} = 2\delta_{ij}\left(I \otimes I - M\right) + 2\sigma_{ji}, \tag{A.5}$$
$$[\sigma_{0i}, M] = 2i\varepsilon_{ijk}\sigma_{jk}, \tag{A.6}$$
$$[\sigma_{i0}, M] = -2i\varepsilon_{ijk}\sigma_{jk},$$
$$\{\sigma_{0i}, M\} = 2\sigma_{i0}, \quad \{\sigma_{i0}, M\} = 2\sigma_{0i}, \tag{A.7}$$

as well as the cubic equations

$$M\sigma_{ij}M = -\sigma_{ij} + 2\sigma_{ji} + 2\delta_{ij}\left(I \otimes I - M\right), \tag{A.8}$$
$$M\sigma_{0i}M = -\sigma_{0i} + 2\sigma_{i0}. \tag{A.9}$$

Using the above formulae and the generalized Euler's exponent identity (B.1) derived in the forthcoming Appendix one can be convinced that

$$e^{i\theta M}\sigma_{ij}e^{-i\theta M} = \cos^2(2\theta)\sigma_{ij} \tag{A.10}$$
$$+ \sin^2(2\theta)\sigma_{ji} + \frac{3}{8}\sin(4\theta)\varepsilon_{ijk}\sigma_{[k0]},$$
$$e^{i\theta M}\sigma_{0i}e^{-i\theta M} = \cos^2(2\theta)\sigma_{0i} \tag{A.11}$$
$$+ \sin^2(2\theta)\sigma_{i0} + \frac{3}{8}\sin(4\theta)\varepsilon_{ijk}\sigma_{jk}.$$

7. THE EULER'S GENERALIZED EXPONENT

Here the following generalization of the Euler's exponent formula[10]:

$$e^{i\theta\boldsymbol{\sigma}\otimes\boldsymbol{\sigma}} = \frac{1}{2}e^{i\theta} \tag{B.1}$$
$$+ \frac{1}{2}e^{-i\theta}\left(\cos(2\theta) + i\boldsymbol{\sigma} \otimes \boldsymbol{\sigma}\sin(2\theta)\right)$$

is proved. The derivation we give is straightforward. Let us introduce the shorthand notation for 4×4 matrix Z

$$\hat{Z} - I \otimes I := \boldsymbol{\sigma} \otimes \boldsymbol{\sigma},$$

with identity 2×2 matrix I. Using the properties of the Pauli matrices $\boldsymbol{\sigma}$ one can easily verify the identity

$$\hat{Z}^2 = 2^2 I \otimes I. \tag{B.2}$$

The left-hand side of (B.1) can be rewritten with aid of the Newton binomial formula and identity (B.2) in the following way

$$e^{i\theta\boldsymbol{\sigma}\otimes\boldsymbol{\sigma}} = \sum_{n=0}^{\infty}\frac{(-i\theta)^n}{n!}(1 - Z)^n \tag{B.3}$$
$$= \sum_{n=0}^{\infty}\frac{(-i\theta)^n}{n!}\sum_k \begin{pmatrix} n \\ k \end{pmatrix} Z^n = \sum_{n=0}^{\infty}\frac{(-i\theta)^n}{n!}$$
$$\times \left[\sum_{r=0}^{[n/2]}\begin{pmatrix} n \\ 2r \end{pmatrix} \cdot 2^{2r} - \hat{Z}\sum_{r=0}^{[(n-1)/2]}\begin{pmatrix} n \\ 2r+1 \end{pmatrix} \cdot 2^{2r}\right].$$

In the last line of (B.3) the notation $[a]$ means the integer part of a number a. Now summing up the finite

[10] Particularly, the Euler's identity $e^{i\pi} = -1$, admits a direct generalization $e^{i\pi\boldsymbol{\sigma}\otimes\boldsymbol{\sigma}} = -I$.

series in (B.3)

$$\sum_{r=0}^{j} \binom{2j}{2r} \cdot 2^{2r} = \frac{1}{2}(1 + 3^{2j}), \qquad \text{(B.4)}$$

$$\sum_{r=0}^{j} \binom{2j+1}{2r} \cdot 2^{2r} = -\frac{1}{2}(1 - 3^{2j+1}), \qquad \text{(B.5)}$$

and

$$\sum_{r=0}^{j} \binom{2j}{2r+1} \cdot 2^{2r} = -\frac{1}{4}(1 - 3^{2j}), \qquad \text{(B.6)}$$

$$\sum_{r=0}^{j} \binom{2j+1}{2r+1} \cdot 2^{2r} = \frac{1}{4}(1 + 3^{2j+1}), \qquad \text{(B.7)}$$

after the rearrangement of the infinite series we find

$$e^{i\theta\boldsymbol{\sigma}\otimes\boldsymbol{\sigma}} = \frac{3}{4}e^{i\theta} + \frac{1}{4}e^{-i3\theta} - \frac{1}{4}\boldsymbol{\sigma}\otimes\boldsymbol{\sigma}\left(e^{-i3\theta} - e^{i\theta}\right). \qquad \text{(B.8)}$$

The identity (B.1) is an equivalent form of this expression.

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