Two-Dimensional Anti-Perron Effect of Changing Different Positive Characteristic Exponents by Higher-Order Perturbations

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Abstract

The two-dimensional anti-Perron effect of changing different positive higher-order smallness characteristic exponents of the linear approximation system by negative ones for a nontrivial solution of the perturbed system is realized.

We consider the two-dimensional differential systems:

(a) the linear system

$$\dot{x} = A(t)x, \quad x \in \mathbb{R}^2, \quad t \geq t_0 \geq 0, \tag{1}$$

with bounded infinitely differentiable coefficients and characteristic exponents

$$\lambda_2(A) \geq \lambda_1(A) > 0;$$

(b) the nonlinear system

$$\dot{y} = A(t)y + f(t,y), \quad y \in \mathbb{R}^2, \quad t \geq t_0, \tag{2}$$

also with an infinitely differentiable, so-called (see, e.g., [1]) m-perturbation $f(t,y)$ of order $m > 1$ smallness in the neighborhood of the origin $y = 0$ and admissible growth outside it

$$\|f(t,y)\| \leq C_f \|y\|^m, \quad C_f = const > 0, \quad y \in \mathbb{R}^2, \quad t \geq t_0. \tag{3}$$

The Perron effect [5], see also [4, pp. 50–51] establishes the existence of two-dimensional systems (1) with all negative exponents and m-perturbations (3) such that the perturbed system (2) has nontrivial solutions with the positive Lyapunov exponents. A series of works due to the authors, including the works written jointly with S. K. Korovin, is devoted to its study. Of greater interest owing to its possible applications is the opposite anti-Perron effect of changing the positive characteristic exponents of the linear approximation (1) by negative ones for (some) nontrivial solutions of the perturbed systems (2) with small perturbations, in particular, m-perturbations (3) of higher order of smallness.

In [2], the anti-Perron effect was realized in the case of coinciding positive characteristic exponents for one nontrivial solution of system (2) with a negative exponent.

In the same case of coinciding positive exponents of the linear approximation, the two-dimensional anti-Perron effect was realized [3] for a larger number of nontrivial solutions with negative exponents of the perturbed system (2) with the corresponding m-perturbation, i.e., for four such solutions.

The question arises on a possible realization of the two-dimensional anti-Perron effect of changing the positive different exponents $\lambda_2(A) > \lambda_1(A) > 0$ of the linear approximation (1) by negative m-perturbations (3) for (some) nontrivial solutions of the perturbed system (2). The theorem below provides a positive answer.

Theorem. *For any parameters*

$$\lambda_2 > \lambda_1 > 0, \quad m > 1, \quad \theta > 1,$$

there exist:

(1) *a two-dimensional linear system* (1) *with bounded infinitely differentiable coefficients and characteristic exponents* $\lambda_i(A) = \lambda_i$, $i = 1, 2$;

(2) *an infinitely differentiable m-perturbation*

$$f(t, y) : [t_0, +\infty) \times \mathbb{R}^2 \to \mathbb{R}^2$$

such that the nonlinear perturbed system (2) *has a solution* $y(t)$ *with the exponent*

$$\lambda[y] = -\frac{(\theta+1)m\theta\lambda_1 + \lambda_2}{m^2\theta^2 - 1}\,.$$

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