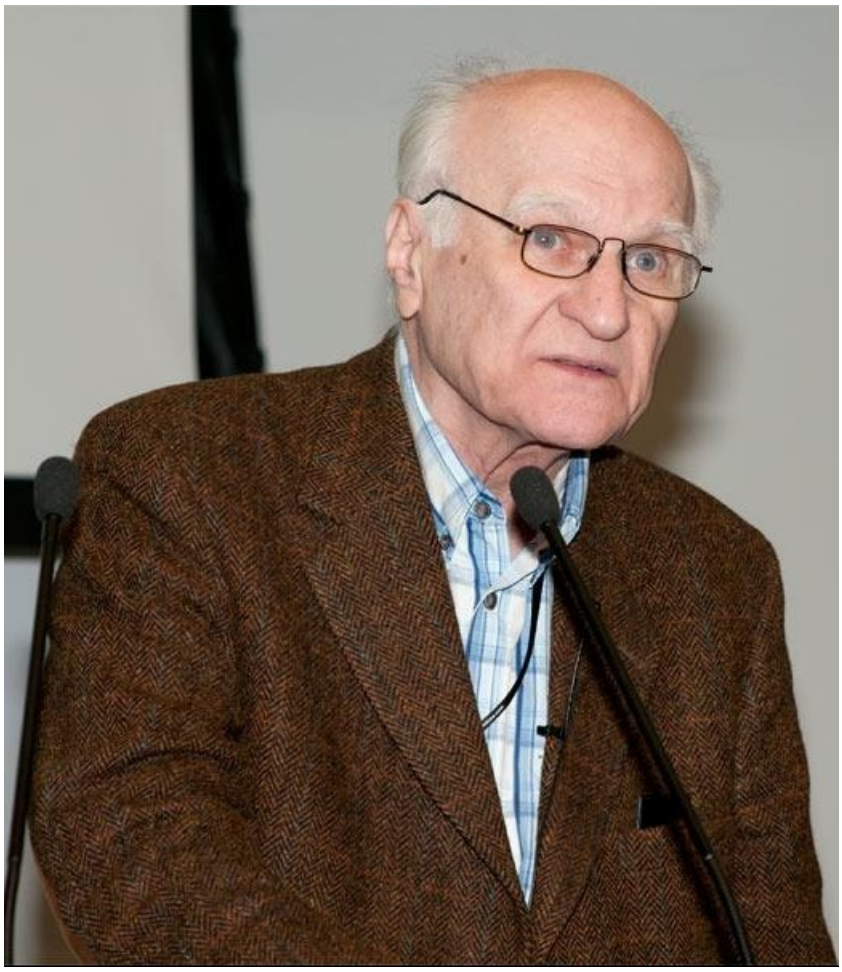
Dedicated to the blessed memory of
Professor Revaz Gamkrelidze



1927 – 2025

In Memory of Professor Revaz Gamkrelidze

An outstanding Georgian mathematician, academician of the Georgian National and Russian Academies of Sciences, Revaz Gamkrelidze was born on February 4, 1927, in Kutaisi (Georgia). His father Valerian Gamkrelidze was one of the prominent figures in the publishing industry in Georgia. After graduating from secondary school, R. Gamkrelidze continued his studies at the Physics and Mathematics Faculty of Tbilisi State University, where he attracted attention from the very beginning as a talented and successful student. All this laid the foundation for R. Gamkrelidze to continue his studies at M. V. Lomonosov Moscow State University. Here he first met with one of the leading figures of mathematical ideas, the Great Russian mathematician, Academician Lev Pontryagin, who left an ineffaceable mark on the creation of scientific interests and the demonstration of the unique talent of young R. Gamkrelidze. The first scientific contribution of R. Gamkrelidze explored Chern's cycles of complex algebraic manifolds, and his next scientific effort focused on nonclassical calculus of variation – specifically, optimal control theory. Together with L. Pontryagin, V. Boltjanskiĭ, and E. Mishchenko, he laid the foundation for the theory of optimal control. R. Gamkrelidze was the first to prove the maximum principle for the linear time-optimal problem – a necessary condition for optimality; he investigated the issue of the existence of optimal control; he introduced the concept of the general state of a system and showed that, for such systems, the maximum principle is a sufficient condition for optimality. It is noteworthy that the Maximun principle, as a hypothesis, was originally proposed by L. Pontryagin and is currently known in the scientific literature as Pontryagin's maximum principle. The same period includes R. Gamkrelidze's fundamental research on optimal control problems with bounded phase coordinates. Achievements of L. Pontryagin school were published in 1961 as a monograph: "L. S. Pontryagin, V. G. Boltjanskiĭ, R. V. Gamkrelidze and E. F. Mishchenko, *The Mathematical Theory of Optimal Processes*. (Russian) Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow, 1961". This work was awarded the State Lenin Prize in 1962. This work is translated into many languages. This book formed the basis for the development of optimal control theory in many countries of the world. Later, R. Gamkrelidze discovered and studied sliding optimal modes. Research of these objects led him to the concepts of quasi-convex set and quasi-convex filter in linear topological spaces, which laid the foundation for the development of the general theory of extremal problems, developed together with G. Kharatishvili. Within this theory, the maximum principle was first proven in integral form. Later, together with A. Agrachev, he constructed an exponential representation of flows and a chronological calculus, obtaining fundamental results in the geometric theory of control. R. Gamkrelidze conducted fruitful scientific and pedagogical activity in various scientific centers around the world. He took part in many international scientific congresses and conferences. He was a member of editorial boards of numerous scientific journals. R. Gamkrelidze's students include many famous scientists from around the world. R. Gamkrelidze founded the Department of Control Theory at Ivane Javakhishvili Tbilisi State University (TSU). He headed the department for a number of years and was also the head of the Department of Differential Equations at the Steklov Institute of Mathematics. R. Gamkrelidze, together with G. Kharatishvili, was recognized as a leading figure in the development of optimal control theory in Georgia and as a successor of the first Georgian mathematician A. Razmadze. The course of lectures delivered by R. Gamkrelidze at TSU later formed the foundation for his monograph *"Principles of Optimal Control Theory*, Mathematical Concepts and Methods in Science and Engineering, Vol. 7. Plenum Press, New York–London, 1978". For this book, R. Gamkrelidze was awarded the A. Razmadze Prize of the Georgian National Academy of Sciences. Revaz Gamkrelidze was awarded the Iv. Javakhishvili

Medal for his long and fruitful scientific, pedagogical and organizational work at Tbilisi State University, and for his significant contribution to the development of the field. Along with the fertile scientific-pedagogical activities, R. Gamkrelidze activity engaged in editorial work. He served as editor-in-chief of many scientific publications, including the abstract journal *"Matematika"*, serial publications: *"Contemporary Mathematics and Its Applications"*, *"Contemporary Mathematics. Fundamental Directions"*, *"Fundamental and Applied Mathematics"*.

Revaz Gamkrelidze passed away on May 5, 2025, his scientific results are included in the Golden Fund of Mathematics. The bright memory of Revaz Gamkrelidze, the kindness and love he sowed, will forever remain in the memories of his students and colleagues.

List of Main Scientific Works by Revaz Gamkrelidze

Monograps

1. The Mathematical Theory of Optimal Processes (with L. S. Pontrjagin, V. G. Boltjanskiĭ, E. F. Mishchenko). (Russian) *Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow*, 1961.
2. Principles of Optimal Control Theory. Mathematical Concepts and Methods in Science and Engineering, Vol. 7. *Plenum Press, New York–London*, 1978.

Scientific Articles

1. Computation of the Chern cycles of algebraic manifolds. (Russian) *Doklady Akad. Nauk SSSR (N.S.)* **90** (1953), 719–722.
2. Chern's cycles of complex algebraic manifolds. (Russian) *Izv. Akad. Nauk SSSR Ser. Mat.* **20** (1956), 685–706.
3. On the theory of optimal processes (with V. G. Boltyanskiĭ, L. S. Pontryagin). (Russian) *Dokl. Akad. Nauk SSSR (N.S.)* **110** (1956), 7–10.
4. On the theory of optimal processes in linear systems. (Russian) *Dokl. Akad. Nauk SSSR (N.S.)* **116** (1957), 9–11.
5. Theory of processes in linear systems which are optimal with respect to rapidity of action. (Russian) *Izv. Akad. Nauk SSSR Ser. Mat.* **22** (1958), 449–474.
6. On the general theory of optimal processes. (Russian) *Dokl. Akad. Nauk SSSR* **123** (1958), 223–226.
7. Optimal control processes for bounded phase coordinates. (Russian) *Izv. Akad. Nauk SSSR Ser. Mat.* **24** (1960), 315–356.
8. The theory of optimal processes. I. The maximum principle (with V. G. Boltjanskiĭ, L. S. Pontryagin). *Amer. Math. Soc. Transl. (2)* **18** (1961), 341–382.
9. On sliding optimal states. (Russian) *Dokl. Akad. Nauk SSSR* **143** (1962), 1243–1245.
10. On a theory of the first variation. (Russian) *Dokl. Akad. Nauk SSSR* **161** (1965), 23–26.
11. On some extremal problems in the theory of differential equations with applications to the theory of optimal control. *J. SIAM Control Ser. A* **3** (1965), 106–128.
12. Extremal problems in linear topological spaces. I (with G. L. Kharatishvili). *Math. Systems Theory* **1** (1967), 229–256.
13. Extremal problems in finite-dimensional spaces. *J. Optim. Theory Appl.* **1** (1967), 173–193.
14. Extremal problems in linear topological spaces (with G. L. Kharatishvili). (Russian) *Izv. Akad. Nauk SSSR Ser. Mat.* **33** (1969), 781–839.

15. Conditions nécessaires du premier ordre dans les problèmes d'extremum (with G. L. Kharatishvili). (French) *Actes du Congrès International des Mathématiciens (Nice, 1970), Tome 3*, pp. 169–176, *Gauthier-Villars Éditeur, Paris*, 1971.
16. Necessary first order conditions, and the axiomatics of extremal problems. (Russian) *Trudy Mat. Inst. Steklov.* **112** (1971), 152–180.
17. A differential game of evasion with nonlinear control (with G. L. Kharatishvili). Translated from the Russian by A. J. Lohwater. *SIAM J. Control* **12** (1974), 332–349.
18. The principle of second order optimality for time-optimal problems (with A. A. Agrachev). (Russian) *Mat. Sb. (N.S.)* **100(142)** (1976), no. 4, 610–643.
19. Exponential representation of flows and a chronological enumeration (with A. A. Agrachev). (Russian) *Mat. Sb. (N.S.)* **107(149)** (1978), no. 4, 467–532.
20. Chronological algebras and nonstationary vector fields (with A. A. Agrachev). (Russian) *Problems in geometry, Vol. 11 (Russian)*, pp. 135–176, 243, Itogi Nauki i Tekhniki, *Akad. Nauk SSSR, Vsesoyuz. Inst. Nauchn. i Tekhn. Inform., Moscow*, 1980; translation in *J. Math. Sci.* **17** (1981), 1650–1675.
21. Differential geometric and group theoretic methods in optimal control theory (with A. A. Agrachev, S. A. Vakhrameev). (Russian) *Problems in geometry, Vol. 14*, 3–56, Itogi Nauki i Tekhniki, *Akad. Nauk SSSR, Vsesoyuz. Inst. Nauchn. i Tekhn. Inform., Moscow*, 1983; translation in *J. Sov. Math.* **28** (1985), no. 2, 145–182.
22. Sliding modes in optimal control theory. (Russian) Topology, ordinary differential equations, dynamical systems. *Trudy Mat. Inst. Steklov.* **169** (1985), 180–193.
23. The index of extremality and quasiextremal controls (with A. A. Agrachev). (Russian) *Dokl. Akad. Nauk SSSR* **284** (1985), no. 4, 777–781.
24. Morse and Maslov indices for the extremals of controllable systems (with A. A. Agrachev). (Russian) *Dokl. Akad. Nauk SSSR* **287** (1986), no. 3, 521–524.
25. Quasi-extremality for controllable systems (with A. A. Agrachev). (Russian) Translated in *J. Soviet Math.* **55** (1991), no. 4, 1849–1864. Itogi Nauki i Tekhniki, *Current problems in mathematics. Newest results, Vol. 35 (Russian)*, 109–134, *Akad. Nauk SSSR, Vsesoyuz. Inst. Nauchn. i Tekhn. Inform., Moscow*, 1989.
26. Ordinary differential equations on vector bundles, and chronological calculus (with A. A. Agrachev, S. A. Vakhrameev). (Russian) Translated in *J. Soviet Math.* **55** (1991), no. 4, 1777–1848. *Itogi Nauki i Tekhniki, Current problems in mathematics. Newest results, Vol. 35 (Russian)*, 3–107, *Akad. Nauk SSSR, Vsesoyuz. Inst. Nauchn. i Tekhn. Inform., Moscow*, 1989.
27. Quadratic mappings and smooth vector functions: Euler characteristics of level sets (with A. A. Agrachev). (Russian) Translated in *J. Soviet Math.* **55** (1991), no. 4, 1892–1928. *Itogi Nauki i Tekhniki, Current problems in mathematics. Newest results, Vol. 35 (Russian)*, 179–239, *Akad. Nauk SSSR, Vsesoyuz. Inst. Nauchn. i Tekhn. Inform., Moscow*, 1989.
28. Local invariants of smooth control systems (with A. A. Agrachev, A. V. Sarychev). *Acta Appl. Math.* **14** (1989), no. 3, 191–237.
29. Symplectic geometry for optimal control (with A. A. Agrachev). *Nonlinear controllability and optimal control*, 263–277, Monogr. Textbooks Pure Appl. Math., 133, *Dekker, New York*, 1990.
30. Symplectic geometry and necessary conditions for optimality (with A. A. Agrachev). (Russian) *Mat. Sb.* **182** (1991), no. 1, 36–54; translation in *Math. USSR-Sb.* **72** (1992), no. 1, 29–45.

31. Volterra series and permutation groups (with A. A. Agrachev). (Russian) Translated in *J. Math. Sci.* **71** (1994), no. 3, 2409–2433. *Itogi Nauki i Tekhniki, Sovrem. Probl. Mat. Nov. Dost., 39, Current problems in mathematics. Newest results, Vol. 39 (Russian)*, 3–40, *Akad. Nauk SSSR, Vsesoyuz. Inst. Nauchn. i Tekhn. Inform., Moscow*, 1991.
32. Local controllability for families of diffeomorphisms (with A. A. Agrachev). *Systems Control Lett.* **20** (1993), no. 1, 67–76.
33. Local controllability and semigroups of diffeomorphisms (with A. A. Agrachev). *Acta Appl. Math.* **32** (1993), no. 1, 1–57.
34. The shuffle product and symmetric groups (with A. A. Agrachev). *Differential equations, dynamical systems, and control science*, 365–382, Lecture Notes in Pure and Appl. Math., 152, *Dekker, New York*, 1994.
35. On the orbits of groups of diffeomorphisms and flows (with A. A. Agrachev). (Russian) *Tr. Mat. Inst. Steklova* **209** (1995), 3–13; translation in *Proc. Steklov Inst. Math.* **209** (1995), 1–10.
36. Feedback-invariant optimal control theory and differential geometry. I. Regular extremals (with A. A. Agrachev). *J. Dynam. Control Systems* **3** (1997), no. 3, 343–389.
37. Symplectic methods for optimization and control (with A. Agrachev). *Geometry of feedback and optimal control*, 19–77, Monogr. Textbooks Pure Appl. Math., 207, *Dekker, New York*, 1998.
38. Vector fields on n-bundles over $2n$-dimensional manifolds (with A. A. Agrachev). (Russian) *Sovrem. Mat. Prilozh.* no. 21, Geom. Zadachi Teor. Upr. (2004), 3–18; translation in *J. Math. Sci. (N.Y.)* **135** (2006), no. 4, 3093–3108.
39. Discovery of the maximum principle. *Mathematical events of the twentieth century*, 85–99, *Springer, Berlin*, 2006.
40. "The Pontryagin derivative" in optimal control. (Russian) *Dokl. Akad. Nauk* **420** (2008), no. 1, 11–13; translation in *Dokl. Math.* **77** (2008), no. 3, 329–331
41. Hamiltonian form of the maximum principle. *Control Cybernet.* **38** (2009), no. 4A, 959–971.
42. The Pontryagin derivative in optimal control. (Russian) *Tr. Mat. Inst. Steklova* **268** (2010), Differentsial'nye Uravneniya i Topologiya. I, 94–99 ISBN: 5-7846-0113-X; 978-5-7846-0113-0; translation in *Proc. Steklov Inst. Math.* **268** (2010), no. 1, 87–92.
43. The geometry of the maximum principle (with A. A. Agrachev). (Russian) *Tr. Mat. Inst. Steklova* **273** (2011), Sovremennye Problemy Matematiki, 5–27 ISBN: 5-7846-0118-0; 978-5-7846-0118-6; translation in *Proc. Steklov Inst. Math.* **273** (2011), no. 1, 1–22.
44. Differential-geometric and invariance properties of the equations of maximum principle (MP). *Geometric control theory and sub-Riemannian geometry*, 167–175, Springer INdAM Ser., 5, *Springer, Cham*, 2014.
45. Dual formulation of the Pontryagin maximum principle in optimal control. Translation of *Tr. Mat. Inst. Steklova* **291** (2015), 69–75; translation in *Proc. Steklov Inst. Math.* **291** (2015), no. 1, 61–67.
46. History of the discovery of the Pontryagin maximum principle. (Russian) *Tr. Mat. Inst. Steklova* **304** (2019), Optimal'noe Upravlenie i Differentsial'nye Uravneniya, 7–14; translation in *Proc. Steklov Inst. Math.* **304** (2019), no. 1, 1–7.