A Version of Anti-Perron Effect of Changing Positive Exponents of Linear Approximation to Negative Ones Under Perturbations of Higher Order of Smallness

N. A. Izobov

Department of Differential Equations, Institute of Mathematics,
National Academy of Sciences of Belarus, Minsk, Belarus
E-mail: izobov@im.bas-net.by

A. V. Il'in

Lomonosov Moscow State University, Moscow, Russia
E-mail: iline@cs.msu.su

We consider the linear differential systems

$$\dot{x} = A(t)x, \;\; x \in R^n, \;\; t \geq t_0, \tag{1}$$

with bounded infinitely differentiable coefficients and characteristic exponents $\lambda_1(A) \leq \cdots \leq \lambda_n(A)$. Along with them, we consider the nonlinear systems

$$\dot{y} = A(t)y + f(t,y), \;\; y \in R^n, \;\; t \geq t_0, \tag{2}$$

with m-perturbations $f(t,y)$ also with infinitely differentiable coefficients of order $m > 1$ smallness in the neighbourhood of the origin $y = 0$ and admissible growth outside it:

$$\|f(t,y)\| \leq C_f \|y\|^m, \;\; m > 1, \;\; C_f = const, \;\; y \in R^n, \;\; t \geq t_0. \tag{3}$$

Perron's effect [6], [5, pp. 50–51] in a two-dimensional case establishes the existence of system (1) with negative exponents and 2 – perturbation (3) such that all nontrivial solutions of the two-dimensional system (2) are infinitely extendable to the right, and a part of them have coinciding positive exponents, and the remaining, nonempty part, has a negative exponent. This effect of changing negative exponents of system (1) to positive for solutions of system (2) is investigated by us (including the joint work with S. K. Korovin) in a cycle of works [1, 2] which are completed by a full description of the sets of positive and negative (and in their absence) exponents of all nontrivial solutions of system (2).

Of greater interest for its possible applications is the anti-Perron effect [3, 4], i.e., the effect of changing all positive exponents of linear approximation (1) to negative ones for the solutions of perturbed systems with small perturbations (with linear exponentially decreasing and tending to zero at infinity; nonlinear of higher order of smallness). Moreover, in [3], the change of exponents

$$\lambda_1(A) > 0 \mapsto \lambda_{n-1}(A+Q) < 0 < \lambda_n(A+Q)$$

is realized by exponentially decreasing linear perturbations $f(t,y) = Q(t)y$ (the case $\lambda_n(A+Q) < 0$ remains open), while in [4] – a complete change of exponents $\lambda_1(A) > 0 \mapsto \lambda_n(A+Q) < 0$ is realized by perturbations $Q(t) \to 0$ for $t \to +\infty$.

In this report, we have realized the following version of the anti-Perron effect of changing the positive exponents of the two-dimensional linear approximation (1) to a negative one for a nontrivial solution of the nonlinear system (2) with m-perturbation (3).

The following theorem is valid.

Theorem. *For any parameters $m > 1$, $\theta > 1$ and $\lambda > 0$ there exist:*

1) *two-dimensional linear system* (1) *with a bounded infinitely differentiable matrix of coefficients $A(t)$ and characteristic exponents $\lambda_1(A) = \lambda_2(A) = \lambda > 0$;*

2) *also infinitely differentiable with respect to its arguments m-perturbation*

$$f(t,y) : [t_0, +\infty) \times R^2 \to R^2,$$

such that the perturbed nonlinear system (2) *has a solution $y(t)$ with the Lyapunov exponent*

$$\lambda[y] = -\lambda \, \frac{\theta+1}{m\theta-1} < 0.$$

References

[1] N. A. Izobov and A. V. Il'in, Construction of an arbitrary Suslin set of positive characteristic exponents in the Perron effect. (Russian) *Differ. Uravn.* **55** (2019), no. 4, 464–472; translation in *Differ. Equ.* **55** (2019), no. 4, 449–457.

[2] N. A. Izobov and A. V. Il'in, Construction of a countable number of different Suslin's sets of characteristic exponents in Perron's effect of their values change. (Russian). *Differentsialnye Uravnenia* **56** (2020), no. 12, 1585–1589.

[3] N. A. Izobov and A. V. Il'in, On the existence of linear differential systems with all positive characteristic exponents of the first approximation and with exponentially decaying perturbations and solutions. (Russian) *Differ. Uravn.* **57** (2021), no. 11, 1450–1457; translation in *Differ. Equ.* **57** (2021), no. 11, 1426–1433.

[4] N. A. Izobov and A. V. Il'in, Linear version of the anti-Perron effect of change of positive characteristic exponents to negative ones. (Russian) *Differ. Uravn.* **58** (2022), no. 11, 1443–1452; translation in *Differ. Equ.* **58** (2022), no. 11, 1439–1449.

[5] G. A. Leonov, *Chaotic Dynamics and Classical Theory of Motion Stability.* (Russian) NITs RKhD, Izhevsk, Moscow, 2006.

[6] O. Perron, Die Stabilitätsfrage bei Differentialgleichungen. (German) *Math. Z.* **32** (1930), no. 1, 703–728.