On Investigation and Approximate Solution of One System of Nonlinear Two-Dimensional Partial Differential Equations

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Systems of nonlinear partial differential equations are describing many real processes. The present note is devoted to one of such mathematical model arising in the investigation of the vein formation in leaves of higher plants and is represented as the two-dimensional nonlinear partial differential system [7]:

$$\begin{aligned} \frac{\partial U}{\partial t} &= \frac{\partial}{\partial x}\Big(V\,\frac{\partial U}{\partial x}\Big) + \frac{\partial}{\partial y}\Big(W\,\frac{\partial U}{\partial y}\Big), \\ \frac{\partial V}{\partial t} &= -V + G\Big(V\,\frac{\partial U}{\partial x}\Big), \\ \frac{\partial W}{\partial t} &= -W + H\Big(W\,\frac{\partial U}{\partial y}\Big), \end{aligned} \tag{1}$$

where $U = U(x,y,t)$, $V = V(x,y,t)$, $W = W(x,y,t)$ are unknown functions defined on the domain $\overline{Q} = \overline{\Omega} \times [0,T] = [0,1] \times [0,1] \times [0,T]$, $T = Const > 0$ and G, H are known functions of their arguments.

In $\overline{Q}$ we consider initial-boundary value problems for (1) and for the following parabolic type regularization of system (1):

$$\begin{aligned} \frac{\partial U_\varepsilon}{\partial t} &= \frac{\partial}{\partial x}\Big(V_\varepsilon\,\frac{\partial U_\varepsilon}{\partial x}\Big) + \frac{\partial}{\partial y}\Big(W_\varepsilon\,\frac{\partial U_\varepsilon}{\partial y}\Big), \\ \frac{\partial V_\varepsilon}{\partial t} &= -V_\varepsilon + G\Big(V_\varepsilon\,\frac{\partial U_\varepsilon}{\partial x}\Big) + \varepsilon\,\frac{\partial^2 V_\varepsilon}{\partial x^2}\,, \\ \frac{\partial W_\varepsilon}{\partial t} &= -W_\varepsilon + H\Big(W_\varepsilon\,\frac{\partial U_\varepsilon}{\partial y}\Big) + \varepsilon\,\frac{\partial^2 W_\varepsilon}{\partial y^2}\,, \end{aligned} \tag{2}$$

with the first type boundary conditions for U, U_ε and the second type boundary conditions for V_ε and W_ε. In (2) we assume that $\varepsilon = Const > 0$.

Some properties of the solutions of initial-boundary problems for systems (1) and (2) are studied.

The convergence of the solution of initial-boundary value problem of the regularized system (2) as $\varepsilon \to 0$ to corresponding solution of model (1) in the norm of the space $L_2(\Omega)$ is discussed.

For building approximate solutions of considered problems two different approaches are used. Both belong to the so-called decomposition methods [8]. The first approach is a decomposition method based on the variable directions difference scheme [1] and the second approach is based on averaged model [8]. The stability and convergence of these schemes are analyzed.

The one-dimensional (1) type system at first has been investigated in [2] and multi-dimensional one in [3,4]. For a brief overview of some research devoted to (1), (2), and relative models we refer to the papers [5,6].

References

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