All Solutions of the Linear Periodic System with the Bounded Reflecting Matrix are Bounded

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We consider the system

$$\frac{dx}{dt} = P(t)x, \quad P(t+2\omega) \equiv P(t), \;\; x \in R^n \tag{1}$$

with the continuous matrix $P(t)$.

The reflecting function of this system has the form $F(t,x) := F(t)x$, where the reflecting matrix $F(t)$ is the solution of the Cauchy problem

$$\frac{\partial F}{\partial t} + F(t)P(t) + P(-t)F = 0, \quad F(0) = E.$$

About that see [1, p. 31]. So if $P(t) + P(-t) \equiv 0$, then $F(t) \equiv E$, where E is identity matrix.

Theorem. *If reflecting matrix $F(t)$ of the periodic system* (1) *is bounded on R, then all solutions of the system* (1) *are bounded on R.*

Example. The equation $\frac{dx}{dt} = x(1 - \cos t)\operatorname{sign} t$ has bounded reflecting matrix $F(t) = 1$. The non-zero solutions of equations are not bounded. But this equation is non-periodic.

References

[1] V. I. Mironenko, *Reflection Function and Periodic Solutions of Differential Equations.* (Russian) "Universitetskoe", Minsk, 1986.