Controllability Linear Differential Systems with Many Inputs by Means of Differential-Algebraic Regulator

Valerii Krakhotko and Georgii Razmyslovich

Belarusian State University, Minsk, Belarus
E-mails: `Krakhotko@bsu.by`; `razmysl@bsu.by`

Consider the control system

$$\dot{x} = Ax + Bu, \quad t \geq 0, \tag{1}$$

with the initial condition $x(0) = x_o$, where $x \in R^n$, and $u \in R^r$, A, B are constant matrices of appropriate sizes, $x_0 \in R^n$.

Definition 1. System (1) is said to be controllable if for each initial condition x_0, there exists a time t_1, $0 < t_1 < +\infty$, and piecewise continuous control $u(t)$, $0 \leq t \leq t_1$, such that the solution $x(t)$, $t \geq 0$, of system (1) satisfies the condition $x(t_1) = 0$.

It is known [3] that for the controllability of system (1) it is necessary and sufficient that

$$\operatorname{rank}(B, AB, \dots, A^{n-1}B) = n. \tag{2}$$

According to the controllability (by Kalman [3]) the input is chosen from the class of piecewise continuous functions. At the same time it is interesting the possibility to choose the control from restricted class.

Let the control be constructed by the input

$$u(t) = Cy(t) \tag{3}$$

of the differential-algebraic system

$$D_0\dot{y}(t) = Dy(t), \ y(0) = y_0, \tag{4}$$

where $y, y_0 \in R^n$, C – $r \times n$-matrix, $D_0 D$ – $n \times n$-matrices.

We say that system (4) is the dynamical regulator for system (1).

Definition 2. System (1) is said to be controllable by dynamical regulator (3) if for each initial condition x_0, there exists a time t_1, $0 < t_1 < +\infty$, and initial condition y_0 of the regulator (4) such that $x(t_1) = 0$.

Theorem. *System* (1) *is controllable by dynamical regulator* (4) *if and only if*

$$\operatorname{rank}(B, AB, \dots, A^{n-1}B) = n$$

and

$$\operatorname{rank}(CD_0^d D_0, CD_0^d K D_0, \dots, CD_0^d K^{n-1} D_0) = n,$$

where D_0^d *– Drazin inverse of* D_0, $K = DD_0^d$.

Список литературы

[1] S. L. Campbell, C. D. Meyer, Jr. and N. J. Rose, Applications of the Drazin inverse to linear systems of differential equations with singular constant coefficients. *SIAM J. Appl. Math.* **31** (1976), no. 3, 411–425.

[2] F. R. Gantmaher, The theory of matrices. 4th ed. *Nauka, Moscow*, 1988.

[3] Р. Е. Калман, Об общей теории систем управления. *Труды Первого Международного конгресса ИФАК*, т. 2, *АН СССР, Москва*, 1961.

[4] V. V. Krakhotko, G. P. Razmyslovich, and V. V. Ignatenko, Controllability linear dynamical system with the help of differential-algebraic regulator. (Russian) *International congress on Computer Science: Information Systems and Technologies (CSIST'2016) Minsk*, 2016, pp. 957–959.