Existence and Asymptotic Behavior (as $t \to +\infty$) of Unboudedly Continuable to the Right Solutions of the Ordinary Differential Equation of the Second Order

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We consider the second order ordinary differential equation of the form:

$$F(t,y,y',y'') = \sum_{k=1}^{n} p_k(t) y^{\alpha_k} |y'|^{\beta_k} |y''|^{\gamma_k} = 0, \tag{1}$$

$n \in \mathbb{N}$, $n \geq 2$, $\alpha_k, \beta_k, \gamma_k \in \mathbb{R}$, $\sum\limits_{k=1}^{n} |\gamma_k| \neq 0$, $p_k \in \mathrm{C}([a;+\infty),\ a>0;\ \mathbb{R})$ $(k=\overline{1,n})$, $p_i(t) \neq 0$ $(i=\overline{1,s}$, $2 \leq s \leq n)$. We investigate the question of the existence and asymptotic behavior (as $t \to +\infty$) of unboudedly continuable to the right solutions (R-solutions) $y(t)$ of equation (1) and the derivatives $y'(t)$, $y''(t)$ of these solutions.

Earlier in [1] we have considered a similar question of the asymptotic behavior of solutions of equation of the form (1) when $\sum\limits_{k=1}^{n} |\gamma_k| = 0$.

The main result is obtained under the assumption that there exists a function $v \in \mathrm{C}^2([t_1;+\infty), t_1 > a; \mathbb{R})$ which possesses the following properties:

(A) $v(t) > 0$, $v''(t) \neq 0$ on $[t_1;+\infty)$, $v(+\infty)$ is equal to 0 or $+\infty$;

(B) $\lim\limits_{t\to+\infty} \dfrac{p_i(t) v^{\alpha_i}(t) |v'(t)|^{\beta_i} |v''(t)|^{\gamma_i}}{p_1(t) v^{\alpha_1}(t) |v'(t)|^{\beta_1} |v''(t)|^{\gamma_1}} = c_i$ $(0 \neq c_i \in \mathbb{R},\ i=\overline{1,s})$,

$$\lim_{t\to+\infty} \frac{p_j(t) v^{\alpha_j}(t) |v'(t)|^{\beta_j} |v''(t)|^{\gamma_j}}{p_1(t) v^{\alpha_1}(t) |v'(t)|^{\beta_1} |v''(t)|^{\gamma_1}} = 0 \ \ (j=\overline{s+1,n});$$

(C) $\exists \lim\limits_{t\to+\infty} \frac{v''(t)v(t)}{(v'(t))^2} = \mu$ $(0 \neq \mu \in \mathbb{R})$.

The following theorem is valid.

Theorem 1. *Let there exist a function $v \in \mathrm{C}^2([t_1;+\infty), t_1 > a; \mathbb{R})$ which possesses the properties* **(A)**–**(C)**. *Then for the R-solution $y(t)$ of the differential equation* (1) *with the asymptotic representation*

$$y^{(k)}(t) \sim v^{(k)}(t) \ \ (k=\overline{0,2}) \tag{2}$$

to exist it is necessary, and if the roots λ_1, λ_2 of the algebraic equation

$$\lambda^2 + \left(1 + \frac{m \sum\limits_{i=1}^{s} (\beta_i + \gamma_i) c_i}{\sum\limits_{i=1}^{s} \gamma_i c_i}\right)\lambda + \frac{m \sum\limits_{i=1}^{s} (\alpha_i + \beta_i + \gamma_i) c_i}{\sum\limits_{i=1}^{s} \gamma_i c_i} = 0$$

have the property $\operatorname{Re}\lambda_k \neq 0$ $(k = 1, 2)$, *then it is also sufficient that* $\sum\limits_{i=1}^{s} c_i = 0$.

Moreover, if in some suburb of $+\infty$ $\operatorname{sign}(\operatorname{Re}\lambda_1) \neq \operatorname{sign}(\operatorname{Re}\lambda_2)$, *then there exists a one-parametric set of* R*-solutions with the asymptotic representation* (2)*; if* $\operatorname{sign}(\operatorname{Re}\lambda_1) = \operatorname{sign}(\operatorname{Re}\lambda_2) \neq \operatorname{sign}(v'(t))$, *then there exists a two-parametric set of* R*-solutions with the asymptotic representation* (2).

This result is obtained using the results from [2, 3].

References

[1] L. Koltsova and A. Kostin, The asymptotic behavior of solutions of monotone type of first-order nonlinear ordinary differential equations, unresolved for the derivative. *Mem. Differential Equations Math. Phys.* **57** (2012), 51–74.

[2] A. V. Kostin, Asymptotics of the regular solutions of nonlinear ordinary differential equations. (Russian) *Differentsial'nye Uravneniya* **23** (1987), No. 3, 524–526, 551.

[3] A. V. Kostin, Stability and asymptotics of quasilinear nonautonomous differential systems. *Uchebnoe posobie, Izd-vo OGU, Odessa* 1984.