Bursting Effect in Neuron Systems

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It is known that self-oscillation (in time) processes in neuron systems exhibit the phenomenon of alternation of pulse bursts (sets of several consecutive intensive spikes) with relatively smooth intervals of membrane potential variation. This phenomenon is referred as *bursting behavior.*

Bursting behavior was studied in numerous works (see, for example, [1–5]). As a rule, the mathematical modeling of this behavior is based on singularly perturbed systems of ordinary differential equations with one slow and two fast variables; under certain conditions such systems may possess stable bursting cycles (periodic motions with bursting behavior). We propose an different approach to the solution of this problem by introducing time delays.

For the mathematical model of an individual neuron we will take the difference-differential equation

$$u' = \lambda\big[f(u(t-h)) - g(u(t-1))\big]u. \qquad (*)$$

Here $u(t) > 0$ is the membrane potential of the neuron, the parameter $\lambda > 0$ (characterizing the time rate of change of electric processes in the system) is assumed to be large, and the parameter $h \in (0,1)$ is fixed. We assume that the functions

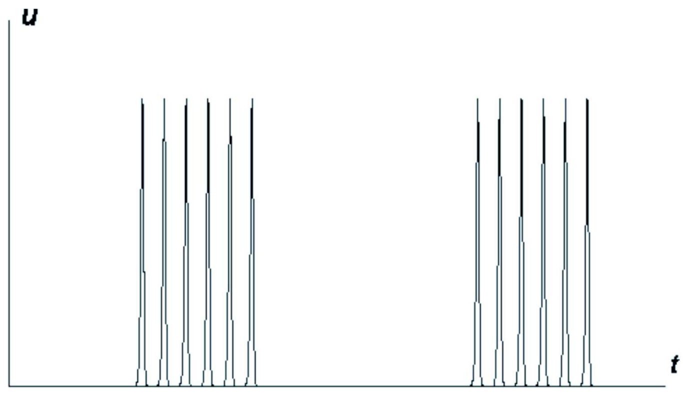
$$f(u), g(u) \in C^1(R^+), \quad R^+ = \big\{u \in R : \geq 0\big\},$$

possess the following properties: $f(0) = 1$, $g(0) = 0$, and as $u \to +\infty$

$$f(u) = -a_0 + O(1/u), \quad uf'(u) = O(1/u),$$
$$g(u) = b_0 + O(1/u), \quad ug'(u) = O(1/u),$$

where a_0 and b_0 are positive constants.

Our main result is as follows. For any fixed natural number n, one can choose the parameters h, a_0, b_0 so that, for all sufficiently large λ, the equation $(*)$ will have an exponentially orbitally stable cycle $u = u^*(t,\lambda)$ of period $T^*(\lambda)$, where $T^*(\lambda)$ tends to a finite limit $T^* > 0$ as $\lambda \to \infty$. On a closed time interval of period length, the function $u^*(t,\lambda)$ has exactly n consecutive asymptotically high (of order $\exp(\lambda h)$)) spikes of duration $\Delta t = [1 + (1/a_0)]h$,while it is asymptotically small at other times. In other words, under such a choice of the parameters h, a_0, b_0 bursting behavior is realized.



The properties of the bursting cycle $u^*(t,\lambda)$ is illustrated on the graph in the plane (t,u) scaled $25 : 1$ for the case $h = 1/26$, $\lambda = 130$ and for the functions

$$f(u) = (1-u)/(1+0.5u), \quad g(u) = 4u/(1+u).$$

References

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