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THE SOLVABILITY OF A NONLINEAR INITIAL BOUNDARY VALUE PROBLEM FOR A BEAM AND THE ALGORITHM OF CONSTRUCTION OF ITS APPROXIMATE SOLUTION

The problem

$$\begin{gathered}
\frac{\partial^2 w}{\partial t^2} = \Big(cd - a + b\int_0^1 \Big(\frac{\partial w}{\partial x}\Big)^2 dx\Big)\frac{\partial^2 w}{\partial x^2} - cd\,\frac{\partial \psi}{\partial x}, \\
\frac{\partial^2 \psi}{\partial t^2} = c\,\frac{\partial^2 \psi}{\partial x^2} - c^2 d\Big(\psi - \frac{\partial w}{\partial x}\Big), \\
0 < x < 1, \quad 0 < t \le T,
\end{gathered} \tag{1}$$

$$\frac{\partial^s w}{\partial t^s}\,(x,0) = w^s(x), \quad \frac{\partial^s \psi}{\partial t^s}\,(x,0) = \psi^s(x), \quad s = 0,1, \tag{2}$$

$$w(0,t) = w(1,t) = 0, \quad \frac{\partial \psi}{\partial x}\,(0,t) = \frac{\partial \psi}{\partial x}\,(1,t) = 0, \tag{3}$$

describing the vibration of a beam with hinged ends, is considered in the Timoshenko model. Here a, b, c, d, T are the given positive constants, $cd - a > 0$, and $w^s(x)$, $\psi^s(x)$ are the given functions, $s = 0,1$.

Assuming that $w^s(x)$ and $\psi^s(x)$ are analytic functions of the forms

$$w^s(x) = \sum_{i=1}^{\infty} a_i^s \sin i\pi x, \quad \psi^s(x) = \frac{b_0^s}{\sqrt{2}} + \sum_{j=1}^{\infty} b_j^s \cos j\pi x, \quad s = 0,1,$$

and applying S. Bernstein's approach, it is proved that there exist functions $w(x,t)$ and $\psi(x,t)$ which are a solution of problem (1)–(3) and which are analytic functions with respect to x for all $0 < t \le T$.

Further, a numerical algorithm of finding a solution of problem (1)–(3) is proposed. This algorithm consists in representing the solution as

$$w_n(x,t) = \sum_{i=1}^{n} w_{ni}(t)\sin i\pi x, \quad \psi_n(x,t) = \frac{\psi_{n0}(t)}{\sqrt{2}} + \sum_{j=1}^{n} \psi_{nj}(t)\cos j\pi x$$

and using Galerkin's method. As a result, we obtain the Cauchy problem for a system of ordinary differential equations with respect to functions $w_{ni}(t)$ and $\psi_{nj}(t)$, $i = 1,2,\dots,n$, $j = 0,1,\dots,n$, which is solved by means of the Crank–Nicholson implicit difference scheme. Therefore on the time grid layers we have to solve a system of nonlinear algebraic equations. For this we use the Picard iteration method. The error of each of three constituent parts of the algorithm is estimated.