G. Khimshiashvili

A. Razmadze Mathematical Institute, Georgian Academy of Sciences
Tbilisi, Georgia

ELLIPTIC BOUNDARY PROBLEMS FOR CAUCHY-RIEMANN SYSTEMS

We deal with boundary value problems for elliptic first order systems of partial differential equations with constant coefficients. The concrete problem which will be addressed, is concerned with the description of those systems which permit elliptic local boundary conditions of Riemann–Hilbert type.

In this context we consider the so-called *generalized Cauchy–Riemann systems* introduced by E. Stein and G. Weiss [3]. They are characterized by the property that each component of every differentiable solution is a harmonic function. For a long time it was known that not all of such systems possess elliptic Riemann-Hilbert problems, but there did not exist neither a reasonable explanation of this phenomenon nor an effective algorithm to verify if a given system has this property. So it was considered as an important problem to give a complete description of those cases when elliptic Riemann–Hilbert problems do exist (see, e.g., [4]).

We describe a partial solution to this problem based on Clifford analysis [2] and topological invariants introduced by P. Baum, R. Douglas and M. Taylor in the framework of operator K-theory [1].

In particular, we will present a comprehensive list of generalized Cauchy–Riemann systems possessing elliptic Riemann–Hilbert problems. This list is complete for systems in the space of any dimension different from 5 and 6.

Some further results related with the index formulae and homotopy classification for elliptic Riemann–Hilbert problems will be also presented.

References

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