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GENERAL SOLUTIONS FOR SOME CLASSES OF LAME GENERALIZED AND CLASSICAL EQUATIONS

Theorem 1. *Let us consider the Lame generalized equation*

$$\alpha(t)\ddot{x} + (\alpha t^2 + \beta t + \gamma)\dot{x} + (\delta t + \eta)x = f(t), \quad \alpha,\, \beta,\, \gamma,\, \delta,\, \eta - \forall\, const, \tag{1}$$

where particular class is defined by

$$\alpha = -\delta, \quad \beta = \frac{\gamma\delta}{\eta} - \eta. \tag{2}$$

General solution of equation (1) *with condition* (2) *has the form*

$$\begin{aligned} x = \Big(t - \frac{\gamma}{\eta}\Big)\Big(c_1 - \int \Big(t - \frac{\gamma}{\eta}\Big)^{-2} e^{-\int \frac{\alpha t^2+\beta t+\gamma}{\alpha(t)}\,dt} \times \\ \times \Big(c_2 - \int e^{\int \frac{\alpha t^2+\beta t+\gamma}{\alpha(t)}\,dt}\Big(t - \frac{\gamma}{\eta}\Big)\frac{f(t)}{\alpha(t)}\,dt\Big)dt\Big). \end{aligned} \tag{3}$$

If the following condition is satisfied

$$\alpha(t) = (t-\tau)(t-\theta)(t-\xi), \quad \tau,\, \theta,\, \xi - \forall\, const, \tag{4}$$

then equation (1) pass into the Lame nonhomogeneous equation (algebraic form).

From Theorem 1 follows

Theorem 2. *If condition* (2) *is satisfied, then the general solution of the Lame nonhomogeneous equation*

$$(t-\tau)(t-\theta)(t-\xi)\ddot{x} + (\alpha t^2 + \beta t + \gamma)\dot{x} + (\delta t + \eta)x = f(t) \tag{5}$$

has form (3), *where* $\alpha(t)$ *has form* (4).

Since

$$\frac{\alpha t^2 + \beta t + \gamma}{(t-\tau)(t-\theta)(t-\xi)} = \frac{k}{t-\tau} + \frac{l}{t-\theta} + \frac{m}{t-\xi},$$

where

$$k = \frac{\alpha\tau^2 + \beta\tau + \gamma}{(\tau-\theta)(\tau-\xi)}, \quad l = \frac{\alpha\theta^2 + \beta\theta + \gamma}{(\theta-\tau)(\theta-\xi)}, \quad m = \frac{\alpha\xi^2 + \beta\xi + \gamma}{(\xi-\tau)(\xi-\theta)},$$

formulae (3) has the form

$$\begin{aligned} x = \Big(t - \frac{\gamma}{\eta}\Big)\Big(c_1 - \int (t-\tau)^{-k}(t-\theta)^{-l}(t-\xi)^{-m}\Big(t - \frac{\gamma}{\eta}\Big)^{-2} \times \\ \times \Big(c_2 - \int (t-\tau)^{k-1}(t-\theta)^{l-1}(t-\xi)^{m-1}\Big(t - \frac{\gamma}{\eta}\Big)f(t)\,dt\Big)\,dt\Big). \end{aligned} \tag{6}$$