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ON THE ASYMPTOTIC STABILITY OF A CLASS OF SYSTEMS OF GENERALIZED LINEAR ORDINARY DIFFERENTIAL EQUATIONS

We study the stability in the Liapunov sense of the system of generalized linear ordinary differential equations

$$dx(t) = dA(t) \cdot x(t) + df(t), \tag{1}$$

where $A : R_+ \to R^{n\times n}$, $f : R_+ \to R^n$ are matrix- and vector-functions with locally bounded variation components.

To a considerable extent, the interest to the theory of generalized ordinary differential equations has been stimulated also by the fact that this theory enables one to investigate systems of ordinary differential, difference and impulse equations.

By a solution of system (1) we understand a vector-function x whose components have bounded total variations on every closed segment from R_+ and $x(t) = x(s) + \int\limits_s^t dA(\tau) \cdot x(\tau)$ $(0 \le s \le t < +\infty)$.

Let $d_1A(t) = A(t) - A(t-)$, $d_2A(t) = A(t+) - A(t)$, $S_0(A)(t) = A(t) - \sum\limits_{0<\tau\le t} d_1A(\tau) - \sum\limits_{0\le\tau<t} d_2A(\tau)$ and $\det(I_n + (-1)^j d_jA(t)) \neq 0$ for $t \in R_+$ $(j = 1, 2)$.

For the concrete matrix-function A we give some effective conditions for its stability.

Theorem. *Let $\alpha_{ik} \in R$ $(i, k = 1, \dots, n)$, and let $\mu_i : R_+ \to R_+$ $(1, \dots, n)$ be nondecreasing functions such that $s_0(\mu_i) \in \widetilde{C}_{loc}(R_+; R_+)$ $(i = 1, \dots, n)$, $\lim\limits_{t\to+\infty} a_0(t) = +\infty$ and $\sigma_i = \liminf\limits_{t\to+\infty} (\alpha_{ii} d_2\mu_i(t)) > -1$ $(i = 1, \dots, n)$, where $a_0(t) \equiv \int\limits_0^t \eta_0(s)ds + \sum\limits_{0<s\le t} \ln(1 - \eta_1(s)) - \sum\limits_{0\le s<t} \ln(1 + \eta_2(s))$, $\eta_0(t) \equiv \min\{|\alpha_{ii}|(s_0(\mu_i)(t))' : i = 1, \dots, n\}$, $\eta_j(t) \equiv \max\{\alpha_{ii} d_j\mu_i(t) :\ i = 1, \dots, n\}$ $(j = 1, 2)$. Then the condition*

$$\alpha_{ii} < 0 \ \ (i = 1, \dots, n), \quad r(H) < 1, \tag{2}$$

where $H = \big(\frac{1-\delta_{ik}}{1+\sigma_i} \frac{|\alpha_{ik}|}{|\alpha_{ii}|}\big)_{i,k=1}^n$, is sufficient for the matrix-function $A(t) = (\alpha_{ik}\mu_i(t))_{i,k=1}^n$ to be asymptotically stable; and if in addition $\alpha_{ik} \ge 0$ $(i \neq k;$ $i, k = 1, \dots, n)$ and $\sum\limits_{k=1, i\neq k}^n \alpha_{ik} d_1\mu_i(t) < \min\{1 - \alpha_{ii}d_1\mu_i(t), |1 + \alpha_{ii}d_1\mu_i(t)|\}$ for $t \in R_+$ $(i = 1, \dots, n)$, then condition (2) is necessary as well.

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