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NECESSARY CONDITIONS OF OPTIMALITY TO LINEAR CONTROL SYSTEMS IN THE PRESENCE OF MIXED AND INTERMEDIATE RESTRICTIONS

We consider the problem

$$I = \int_{t_0}^{t_1} f_0\big(x(t), u(t)\big)\, dt \to \inf \tag{1}$$

under the restrictions

$$\dot{x}(t) = f\big(x(t), u(t)\big), \tag{2}$$

$$g\big(x(t), u(t)\big) \le 0, \tag{3}$$

$$q\big(x(t_{(1)}), \dots, x(t_{(N)})\big) = 0, \tag{4}$$

where $[t_0, t_1]$ is e fixed segment from R^1, $x = x(t) \in W^n_{1,1}[t_0, t_1]$, $u = u(t) \in L^r_1[t_0, t_1]$, $f = (f_1, ..., f_n)'$, $g = (g_1, ..., g_m)'$, $q = (q_1, ..., q_s)'$, $t_{(p)} \in [t_0, t_1]$, $p = \overline{1, N}$,$t_{(1)} = t_0, t_{(N)} = t_1$. The functions f, g, q are linear with respect to all arguments. Moreover, the restrictions (2), (3) are fulfilled almost everywhere on $[t_0, t_1]$ and the restrictions (3) satisfy the conditions of generality, i.e., for any (x, u) satisfying (3) the system of vectors $\mathrm{grad}_u g^j(x, u), j \in J(x, u)$ is linearly independent. Here by $J(x, u)$ we denote the set of such indices j $(j = \overline{1, m})$ for which $g^j(x, u) = 0$.

Since the restrictions (3) are equivalent to equality type restrictions $y^2 + g(x, u) = 0$,where $y = y(t) \in L^m_2[t_0, t_1]$, $y^2 = ((y^1)^2, \dots, (y^m)^2)'$, therefore it is easy to see that problem (1)–(4) is a particular case of the general extremal problem considered in [1]. Using Theorem 1 from [1] we obtain the necessary conditions of optimality for problem (1)–(4).

References

1. Z. Tsintsadze, The Lagrange principle of taking restrictions off and its application to linear optimal control problems in the presence of mixed restrictions and delays. *Mem. Differential Equations Math. Phys.* **11**(1997), 105-128.