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A NONLINEAR VERSION OF ASGEIRSSONIAN MEAN VALUE PROPERTY AND ITS APPLICATION TO CHARACTERISTIC PROBLEMS

A nonlinear version of known Asgeirsson mean value property is considered for the special class of second order quasilinear hyperbolic equations with parabolic degeneracy, whose general solutions are represented by means of superposition of two arbitrary functions. In the simplest case of the equation

$$2y(u_x - 2y)u_{xx} + (u_y - 2yu_x - 2y)u_{xy} - u_x u_{yy} = 2u_x(u_x - 1), \quad (1)$$

the general solution is

$$u = y^2 + f[y + g(x - y^2)]. \quad (2)$$

Characteristic families of (1) are represented by relations: $x - y^2 = c$, $u - y^2 = c$.

The analogue of Asgeirsson principle for nonlinear equations is as follows: *the sums of ordinates on the opposite vertexes of arbitrary characteristic quadrangle are equal.* This principle can be used for investigation of different nonlinear hyperbolic problems for equation (1), including Goursat characteristic problem, which in this case means:

to define the regular solution $u(x, y)$ of equation (1) simultaneously with its domain of definition if it satisfies the condition

$$u|_{x=y^2} = \varphi(y), \quad 0 \le y \le a, \quad \varphi \in C^2[0, a] \quad (3)$$

and the arc of the curve

$$x = \psi(y), \;\; 0 \le y \le b \quad (4)$$

is characteristic along this solution.

Using the mean value property, under the condition $\psi'(y) - 2y > 0$, characteristic of $u - y^2 = c$ and corresponding solution are constructed in the explicit form:

$$x = \psi\{y - y_0 + \Psi(x_0 - y_0^2)\} - 2y\{y_0 + \Psi(x_0 - y_0^2)\} + \{y_0 + \Psi(x_0 - y_0^2)\}^2, \quad (5)$$

$$u = y^2 + \varphi\{y - \Psi(x - y^2)\} - \{y - \Psi(x - y^2)\}^2, \quad (6)$$

where (x_0, y_0) are coordinates of an arbitrary point between characteristic curves $x = y^2$, $x = \psi(y)$, $x - y^2 = \psi(b) - b^2$ and $x = \psi(y - a) + 2ay - a^2$. Characteristic quadrangle bounded by these characteristic arcs in compliance with data (3), (4) is the definition domain of solution (6).

Theorem. *If the relation $\psi(y) - y^2 = z$ has only one solution as the functional equation with respect to y, then problem* (1), (3), (4) *is uniquely solvable.*