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ON A MULTI-POINT BOUNDARY VALUE PROBLEM FOR HIGHER ORDER SINGULAR DIFFERENTIAL EQUATIONS

Optimal sufficient conditions are found for the solvability and unique solvability of the boundary value problem

$$u^{(n)} = f(t, u, \cdots, u^{(n-1)}), \tag{1}$$

$$u^{(i-1)}(t_0)=0 \ (i=1,\dots,n-1), \ \ \sum_{k=1}^{n}\alpha_k u^{(k-1)}(a_k)+\sum_{k=1}^{n}\beta_k u^{(k-1)}(b_k)=0. \tag{2}$$

Here $n \geq 2$, $a \leq a_k < t_0 < b_k \leq b$ $(k = 1,\dots,n)$, $(-1)^{n-k}\alpha_k \geq 0$, $\beta_k \geq 0$ $(k = 1,\dots,n)$, $\sum\limits_{k=1}^{n}(|\alpha_k| + \beta_k) > 0$, and $f : I_{t_0} \times R^n \to R$ is a measurable in the first argument and continuous in the last n arguments function, $I_{t_0} = [a, b] \setminus \{t_0\}$. Moreover, the function f, generally speaking, is nonintegrable in the first argument on $[a, b]$ as it has the singularity at the point t_0.

In particular, the following theorem is proved.

Theorem. *Let there exist measurable functions $g_k, \overline{g}_k : I_{t_0} \to [0, +\infty[$ and an integrable function $g : [a, b] \to [0, +\infty[$ such that*

$$\int_a^b |t - t_0|^{n-k}\overline{g}_k(t)\,dt < +\infty \quad (k = 1,\dots,n),$$

$$\begin{gathered}\sum_{k=1}^{n}\frac{1}{(n-k)!}\int_a^{t_0}(t_0 - t)^{n-k}g_k(t)\,dt \leq 1,\\ \sum_{k=1}^{n}\frac{1}{(n-k)!}\int_{t_0}^{b}(t - t_0)^{n-k}g_k(t)\,dt \leq 1,\end{gathered} \tag{3}$$

and on the set $I_{t_0} \times R^n$ the following inequalities are satisfied:

$$f(t, x_1, \cdots, x_n)\,\mathrm{sgn}[(t - t_0)^{n-1}x_1] \geq -\sum_{k=1}^{n} g_k(t)|x_k| - g(t),$$

$$|f(t, x_1, \cdots, x_n)| \leq \sum_{k=1}^{n}\overline{g}_k(t)|x_k| + g(t).$$

Then problem $(1), (2)$ *has at least one solution.*

Note that in the right-hand sides of inequalities (3) we cannot write down $1 + \varepsilon$ instead of 1 no matter how small $\varepsilon > 0$ would be.

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