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A GENERAL REPRESENTATION FORMULA FOR A SOLUTION OF THE SYSTEM OF STATICS OF ELASTIC MIXTURES

The system of differential equations of statics of the linear theory of elastic mixtures (in the case of two-dimensions) reads as follows

$$\begin{aligned} &a_1\Delta u' + b_1 \operatorname{grad}\operatorname{div} u' + c\Delta u'' + d \operatorname{grad}\operatorname{div} u'' = 0,\\ &c\Delta u' + d \operatorname{grad}\operatorname{div} u' + a_2\Delta u'' + b_2 \operatorname{grad}\operatorname{div} u'' = 0, \end{aligned} \tag{1}$$

where $u' = (u_1', u_2')$ and $u'' = (u_1'', u_2'')$ are the partial displacement vectors,

$$a_1 = \mu_1 - \lambda_5,\;\; a_2 = \mu_2 - \lambda_5,\;\; c = \mu_3 + \lambda_5,\;\; b_1 = \mu_1 + \lambda_5 + \lambda_1 - \frac{\rho_2}{\rho}\alpha_2',$$

$$b_2 = \mu_2 + \lambda_2 + \lambda_5 + \frac{\rho_1}{\rho}\alpha_2',\;\; d = \mu_3 + \lambda_3 - \lambda_5 - \frac{\rho_1}{\rho}\alpha_2',\;\; \alpha_2' = \lambda_3 - \lambda_4,$$

$\rho = \rho_1 + \rho_2$, ρ_1 and ρ_2 are the partial densities, $\lambda_1, \lambda_2, \dots, \lambda_5$, μ_1, μ_2, μ_3 are elastic constants.

The following result is proved: a solution to the system (1) can be represented by means of four harmonic functions

$$\begin{aligned} &u'(x) = \operatorname{grad}\Phi_1(x) + \\ &+ r^2 \operatorname{grad} \Big[\;\alpha_1 + \frac{1}{2} \;\; r\frac{\partial}{\partial r} + 2 \;\; - 1\Big]\Phi_2(x) + \beta_1 \;\; r\frac{\partial}{\partial r} + 2 \;\; \Phi_3(x) \;\; - \\ &- xr\frac{\partial}{\partial r} \;\; (2\alpha_1 - 1)\Phi_2(x) + 2\beta_1\Phi_3(x) \;\; + \chi'(x),\\ &u''(x) = \operatorname{grad}\Phi_4(x) + \\ &+ r^2 \operatorname{grad} \;\; \alpha_2 \;\; r\frac{\partial}{\partial r} + 2 \;\; \Phi_2(x) + \Big[\;\beta_2 + \frac{1}{2} \;\; r\frac{\partial}{\partial r} + 2 \;\; - 1\Big]\Phi_3(x) \;\; - \\ &- xr\frac{\partial}{\partial r} \;\; 2\alpha_2\Phi_2(x) + (2\beta_2 - 1)\Phi_3(x) \;\; + \chi''(x), \end{aligned}$$

where $\Delta\Phi_j(x) = 0$, $j = 1, 2, 3, 4$, $r\frac{\partial}{\partial r} = (x \cdot \operatorname{grad})$,

$$\chi'(x) = \begin{cases} A_0x + B_0\widetilde{x}, & x \in \Omega^+,\\ 1/r^2(A_0x + B_0\widetilde{x}), & x \in \Omega^-, \end{cases} \quad \chi''(x) = \begin{cases} C_0x + D_0\widetilde{x}, & x \in \Omega^+,\\ 1/r^2(C_0x + D_0\widetilde{x}), & x \in \Omega^-, \end{cases}$$

$x = (x_1, x_2)$, $\widetilde{x} = (-x_2, x_1)$, A_0, B_0, C_0, and D_0 are arbitrary constants, Ω^+ is a bounded domain, $\Omega^- = \mathbb{R}^2 \setminus \overline{\Omega^+}$,

$$\alpha_1 = \frac{1}{2\Delta_1}(cd - b_1a_2 - \Delta_1),\;\; \beta_1 = \frac{1}{2\Delta_1}(cb_2 - a_2d),$$

$$\alpha_2 = \frac{1}{2\Delta_1}(cb_1 - a_1d),\;\; \beta_2 = \frac{1}{2\Delta_1}(cd - a_1b_2 - \Delta_1),\;\; \Delta_1 = a_1a_2 - c^2.$$