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A MAXIMUM PRINCIPLE IN THE VARIATIONAL INEQUALITIES

In the present paper a week maximum principle for a bilinear, elliptic, nonsymmetric and coercive form, with the L^∞ coeficients is given. The result is applied to prove the maximum principle for the variational inequalities.

Let $f \in L_2(\Omega)$, $h \in H^1(\Omega)$, $H^1(\Omega)$ be the real Sobolev space, $K_1 = \{ v \in H^1(\Omega) : \ v \geq h\}$, and $a(u,v)$ be the above-mentioned bilinear form. The problem is to find $u \in K_1$, such that

$$a(u, v-u) \geq \int_\Omega f \cdot (v-u)dx, \ \ \forall v \in K_1. \tag{1}$$

In some restrictions there is proved the equality es sup$u$=es sup$h$ for this inequality. This fact give us an opportunity to locate the set of coincidence $(u = h)$ for some concrete cases. Inequality (1) is also considered on the set $K_2 = \{v \in H^1(\Omega) : \ \phi \geq v \geq \psi\}$, with ϕ, $\psi \in H^1(\Omega)$. Based on the above week maximum principle, for some cases there is proved the equalities es sup$u$=es supψ and es inf$u$=es infϕ.

With the help of the maximum principle, written for the variational inequality in the set K_1, we prove the following result

$$a(u-u_\lambda, \Psi) \geq 0, \ \ \forall \Psi \in H^1(\Omega), \ \ \Psi \geq 0. \tag{2}$$

Here $\lambda \in R^+$, u and u_λ are the solutions of the inequality (1) in the sets K_1 and $K_1^\lambda = \{v \in H^1(\Omega) : \ v \geq h - \lambda\}$.

Finally, inequality (1) is considered with the implicit obstacle, i.e., in the set $K_3 = \{v \in H^1(\Omega) : \ v \geq h - a(u, \Phi)\}$, with $\Phi \in H^1(\Omega)$, $\Phi \geq 0$. With the help of (2), the unique solvability of this problem is proved.