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ON SOME NONLINEAR PROBLEMS IN THE THEORY OF ELASTIC RODS

The problem of the bifurcation of equilibrated forms of the axis of the rod subjected to the joint action of axial compression force and distributed force of given intensity is investigated, taking into account tension-compression and displacement. The problem is described by a boundary value problem for a second order nonlinear ordinary differential equation depending on two real parameters (see [1], §8).

Questions on the existence of bifurcation points and continuous solution-branches of this boundary value problem are studied when axial compression force is proportional to transverse load. Functionals are constructed to find bifurcation points which are critical values of forces. Asymptotic representations of small real solutions of this problem are given, which describe an approximate form of post-critical deformation of the rod.

In studying the problem of small bendings of rods of variable rigidity, the first eigenvalue (the critical value of load applied to the free end of the rod) of the linearized problem is expressed in terms of an energy functional (see, for instance, [2]). Of essential technical interest is the problem on a conditional maximum of the first eigenvalue with respect to rod rigidity. These questions led to the study of extreme properties of the first eigenvalue of the Sturm–Liouville operator with respect to the coefficients of a differential equation. The first eigenvalue is considered as the functional of these coefficients. The problem of a functional extremum is solved in the conditions of isoperimetric connections. Necessary and sufficient conditions are obtained for the first eigenvalue to reach its conditional extremum.

REFERENCES

1. V. V. Eliseyev, Mechanics of elastic rods. (Russian) *Sankt-Petersburg, SPB State Tech. Univ.*, 1994.
2. D. P. Zeragia, *Differentsial'nye Uravneniya* **7**(1971), No. 9, 1603–1610.