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SOME PROBLEMS OF THE THEORY OF ELASTICITY FOR A NONHOMOGENEOUS PLANE WITH RECTILINEAR CRACKS

An elastic medium occupying the entire infinite plane of the variable $z = x + iy$ and weakened by rectilinear cracks is considered in the conditions of plane stressed state. The nonhomogeneous medium is characterized by the variable Poisson coefficient (the shift modulus is a constant value): $\sigma(y) = \sum_{k=0}^{m}(A_k \cos k\alpha y + B_k \sin k\alpha y)$, where A_k $(k = 0, 1, 2, \dots, n)$, α are real-valued constants. We proceed from the complex representation formulas [1]:

$$X_x + Y_y = 2 \ \Phi(z) + \overline{\Phi(z)} \ \sum_{k=0}^{n} \ A_k \cos k(\overline{a}_0 z + a_0 \overline{z}) + B_k \sin k(\overline{a}_0 z + a_0 \overline{z}) \ ,$$

$$Y_y - X_x - 2iX_y = 2\Big\{A_0 z \overline{\Phi'(z)} + \overline{\Psi(z)} - \frac{\overline{\Phi'(z)}}{a_0} \sum_{k=1}^{n} \frac{1}{k} \ A_k \sin k(\overline{a}_0 z + a_0 \overline{z}) -$$

$$-B_k \cos k(\overline{a}_0 z + a_0 \overline{z}) \ - \overline{\Phi(z)} \sum_{k=1}^{n} \ A_k \cos k(\overline{a}_0 z + a_0 \overline{z}) + B_k \sin k(\overline{a}_0 z + a_0 \overline{z}) \ - \qquad (a)$$

$$-a_0 \sum_{k=1}^{n} k G_k^{*'}(z) \ A_k \sin k(\overline{a}_0 z + a_0 \overline{z}) - B_k \cos k(\overline{a}_0 z + a_0 \overline{z}) \ -$$

$$-a_0^2 \sum_{k=1}^{n} k^2 G_k^{*}(z) \ A_k \cos k(\overline{a}_0 z + a_0 \overline{z}) + B_k \sin k(\overline{a}_0 z + a_0 \overline{z}) \ \Big\},$$

where $a_0 = \frac{\alpha}{2} i$, $\Phi(z) = \prod_{j=1}^{k} \ \frac{d^2}{dz^2} + j^2 a_0^2 \ G_k(z)$, $G_k^*(z) = \prod_{j=2}^{k} \ (\frac{d^2}{dz^2} + (j-1)^2 a_0^2 \ G_k(z)$, $G_1^*(z) = G_1(z)$, $G_p(z) = G_{p+1}(z) + (p+1)^2 a_0^2 G_{p+1}(z)$ $(k, p = 1, 2, \dots, n)$; X_x, Y_y, X_y are stress components, $C_k(z)$ $(k = 1, 2, \dots, n)$, $\Psi(z)$ are complex potentials on the plane z.

Let us assume that the considered cracks lie along the ox-axis of the unbounded plane z and consider the case, where load is applied only to the crack surfaces, since other cases of load application can be reduced to this case by superposition. By formulas (a) and the method used in [2, §120], [3, Ch. 3, Section III, A, B], the problem is reduced to the Riemann–Hilbert boundary value problem, an explicit solution is found, and the stress intensity coefficient is determined near the crack ends.

References

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