

E. Gordadze

A. Razmadze Mathematical Institute, Georgian Academy of Sciences
Tbilisi, Georgia

ON THE PROBLEM OF LINEAR CONJUGATION IN THE CASE OF NON-SMOOTH LINES

1. Let S_Γ be a singular operator, $S_\Gamma \varphi = (\pi i)^{-1} \int_\Gamma \varphi(t)(t - \tau)^{-1} dt$. If S_Γ is bounded in $L_p(\Gamma)$, then we write $\Gamma \in R$. If the operator ρS_ρ is also bounded in $L_p(\Gamma)$, where $\rho(t)$ is a positive measurable function, then we write $\rho \in W_p(\Gamma)$. Denote by Γ_t a continuous on Γ arc (which open if t is an exterior point, and closed if Γ is unclosed, and t is one of the end points), such that $t \in \Gamma_t$.

Theorem. *If Γ is the Jordan line, $\Gamma \in R$, and for every $t \in \bar{\Gamma}$ there exists Γ_t such that $\rho \in W_p(\Gamma_t)$, then $\rho \in W_p(\Gamma)$.*

2. Consider the boundary value problem of linear conjugation in the Hardy–Smirnov class $E_p(D_\Gamma^\pm)$, $p > 1$. Suppose that $\Gamma \in R$, Γ is the Jordan line and $\Gamma = \bigcup_{k=1}^n \Gamma_{a_k a_{k+1}}$, $a_{n+1} = a_1$ (Γ_{ab} denotes the arc with the ends a and b which is continuous from a to b). Assume that $\Gamma_{a_k a_{k+1}}$ can be supplemented up to the closed contour $\Gamma_k^0 \in R$ for which the problem under Simonenko's conditions with the zero index has the unique solution. Contours of the type Γ_k^0 have been considered by many authors, however, the existence of cusps was excluded everywhere. It is evident that the cusps and more complicated cases may occur for the lines $\Gamma = \bigcup_{k=1}^n \Gamma_{a_k a_{k+1}}$ at the points a_k . We introduce obvious enough additional restrictions on $G(t)$ in the neighborhood of a_k which allow one to obtain the classical result for the above-mentioned lines. As an example, we can name both the piecewise smooth lines and the Radon lines.