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FORMULAE FOR THE GENERAL SOLUTIONS FOR SOME CLASSES OF LINEAR NONHOMOGENEOUS DIFFERENTIAL EQUATIONS OF SECOND ORDER WITH VARIABLE COEFFICIENTS

Let us consider the equation

$$\alpha(t)x'' + \beta(t)x' + \gamma(t)x = \delta(t), \quad t_1 < t < t_2, \tag{1}$$

where α, β, γ, δ are given continuous functions.

The function is called possible if the above operations are correct.

Theorem 1. *If the following conditions are satisfied*

$$\alpha(t) \equiv 1, \quad \beta(t) \equiv 0, \quad \gamma(t) = -\frac{\eta''(t)}{\eta(t)}\,, \quad t_1 < t < t_2, \tag{2}$$

where η is any possible function, then the general solution of the equation (1) *has the following form*

$$x = \eta \quad c_1 + \int \frac{1}{\eta^2} \quad c_2 + \int \eta\delta \, dt \quad dt \quad , \quad t_1 < t < t_2. \tag{3}$$

Let

$$\eta = e^{\int \xi(t)\, dt}, \quad t_1 < t < t_2, \tag{4}$$

where ξ is any possible function, then from the Theorem 1 follows

Theorem 2. *If the following conditions are satisfied $\alpha(t) \equiv 1$, $\beta(t) \equiv 0$, $\gamma(t) = -(\xi'(t) + \xi^2(t))$, $t_1 < t < t_2$, then the general solution of the equation* (1) *is given by* (3), *where the function η is determined by* (4).

There also is much general result.

Theorem 3. *If the following conditions are satisfied*

$$\begin{cases} ab = \alpha(t) \\ ab' + qb + ah = \beta(t) \\ ah' + qh = \gamma(t), \quad t_1 < t < t_2 \end{cases},$$

where $a = a(t)$, $b = b(t)$, $q = q(t)$, $h = h(t)$ are any possible functions, then the general solution of the equation (1) *is represented in finite terms by* (3).

Finally, we note that Theorem 2 generalizes the results of M. Elshin (*Bull. Acad. Sci. USSR*, **XVIII**(1938)) and has important applications. For example if $\xi(t) = Ae^{\omega t} + B$, where A, B, ω are constants, then from the equation (1), assuming $\alpha(t) \equiv 1$ and $\beta(t) \equiv 0$, one gets generalized Hille's type equation.