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ON ONE PROPERTY OF A 2-RANK HOLOMORPHIC VECTOR BUNDLE ON THE RIEMANN SPHERE

Let $S = \{s_1, s_2, \ldots, s_m\}$ be a set of marked points on $\mathbf{CP}^1$ and

$$df = \Big(\sum_{i=1}^{m} \frac{A_i}{z - s_i}\, dz\Big) f \tag{1}$$

be the system of ODE's which is induced by the representation

$$\rho : \pi_1(\mathbf{CP^1} \backslash \mathbf{S}, \mathbf{z_0}) \to \mathbf{GL(n, C)},$$

where the matrices A_i satisfy the condition $\sum\limits_{i=1}^{m} A_i = 0$.

For every Fuchsian equation there exists a Fuchsian system (1), which has the same singular points and monodromy. In particular, for the hypergeometric equation

$$y'' + \frac{\gamma + (\alpha + \beta + 1)}{z(z-1)} y' - \frac{\alpha\beta y}{z(z-1)} = 0$$

the aforementioned system will be:

$$df = \left(\begin{pmatrix} 0 & 0 \\ -\alpha\beta & -\gamma \end{pmatrix} \frac{dz}{z} + \begin{pmatrix} 0 & 1 \\ 0 & \gamma - (\alpha + \beta) \end{pmatrix} \frac{dz}{z-1} \right) f \tag{2}$$

Fix a new representation ρ for any Riemann data $(\mathbf{CP^1}, \mathbf{S}, \rho)$ and denote by Ω_ρ the set of Fuchsian systems corresponding to this data. The number $\gamma_\rho = \min_{\Omega_\rho} \gamma_\omega$ is called the Fuchsian weight for the representation ρ.

Let $\mathbf{E} \to \mathbf{CP^1}$ be the holomorphic vector bundle induced by the representation ρ and (k_1, k_2) be its splitting type. Then $\gamma_\rho = k_1 - k_2$.

Every rank two holomorphic bundle on $\mathbf{C}P^1$ is holomorphically equivalent to any bundle $\mathbf{F} \to \mathbf{CP^1}$, which is obtained by the extension of the bundle induced by an irreducible representation. So, for every rank 2 holomorphic bundle there exists an irreducible connexion, which is holomorphic except for the finite number n_ω of points, where it has simple poles. Denote by $\Omega^{\rm irr}$ the space of irreducible Fuchsian connexions. Let $p = \min_{\omega \in \Omega^{\rm irr}} n_\omega$. Then the identity $p = k_1 - k_2 + 2$ is satisfied.

From this follows a criterion of stability of such bundles.

Proposition. *A rank two vector bundle* $\mathbf{F} \to \mathbf{CP^1}$ *is stable if and only if it is induced by the system* (2).