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OPTIMIZATION OF LINEAR CONTROLLED SYSTEMS WITH RESTRICTIONS OF "NARROW PLACE" TYPE

Consider an industrial process (see [1]), which is described with the linear system of differential equations

$$\dot{x}_1 = u_1, \quad \dot{x}_2 = u_2, \quad \dot{x}_3 = u_3 - b_1 u_1 - b_2 u_2 - b_4 u_4, \quad \dot{x}_4 = u_4 \tag{1}$$

under the initial conditions

$$x_1(0) = c_1, \quad x_2(0) = c_2 > 0, \quad x_3(0) = c_3, \quad x_4(0) = c_4 > 0, \tag{2}$$

where $x_1(t)$ is the reserve of motor-car in moment t, $x_2(t)$ is the industrial power of motor-car branch in t, $x_3(t)$ is the reserve of steel in moment t, $x_4(t)$ is the industrial power of steelfoundry branch in moment t, b_i, c_i are given constants and $u_i = u_i(t)$, $i = \overline{1,4}$, are the control functions.

We consider the optimal control problem for this process in the form:

$$x_1(T) \to \mathrm{Sup} \tag{3}$$

under the restrictions

$$u_1 - x_2 \leq 0, \quad u_3 - x_4 \leq 0, \quad u_{j-2} \leq 0, \quad j = 3,4,5,6, \quad x_3 > 0 \tag{4}$$

and conjunctions (1), (2). Here T is fixed, u_1 is a speed of motor-car production, u_2 is a speed of industrial power rising of motor-car branch, u_3 is a speed of steelproduction, u_4 is a speed of industrial power rising of steelfoundry branch. In different from general problems, which are considered in common theory from [2], the given problem with restrictions of "narrow place" type is characterised.

Let $x_i(t)$, $i = \overline{1,4}$, be absolutely continuous, $u_i(t)$, $i = \overline{1,4}$, be integrable functions on $[0,T]$, and the conditions (1), (4) be fulfilled almost everywhere on $[0,T]$, then using the Theorem 1 from [3], we obtain the necessary conditions of optimality for problem (1)–(4).

Detailed analysis of the behaviour of system (1)–(4) when satisfying the necessary conditions of optimality enables one to optimize this system for different corelations between parameters b_1, b_2, b_4 and T. For example, if $T < b_4$, then the optimal is such a behaviour of the system at which the industrial power of both branches remain constant and are used maximally.

References

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