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BOUNDARY VALUE PROBLEMS FOR TWO-DIMENSIONAL DELAYED DIFFERENTIAL SYSTEMS

The question on the solvability of the boundary value problem

$$\frac{du_i(t)}{dt} = f_i\big(t, u_1(\tau_1(t)), u_2(\tau_2(t))\big) \ \ (i=1,2), \tag{1}$$

$$u_1(a) = \alpha u_2(a), \quad \beta_1 u_1(b) + \beta_2 u_2(b) = \gamma \tag{2}$$

is investigated. Here $f_i : [a,b] \times \mathbb{R}_+^2 \to \mathbb{R}_+$ and $\tau_i : [a,b] \to [a,b]$ $(i=1,2)$ are continuous functions, $\mathbb{R}_+ = [0, +\infty[$, $\alpha \in \mathbb{R}_+$, $\beta_i \in \mathbb{R}_+$ $(i=1,2)$, $\beta_1 + \beta_2 > 0$, $\gamma > 0$, and

$$\tau_i(t) \le t, \quad f_i(t,0,0) = 0 \ \text{ for } \ a \le t \le b \ \ (i=1,2).$$

In particular, the following theorem is proved.

Theorem. *For the solvability of problem* $(1), (2)$ *it is sufficient one of the following five conditions to be fulfilled:*

(i) $\beta_1 > 0$, $\beta_2 > 0$*;*

(ii) $\beta_1 = 0$, $\beta_2 = 1$*, and there exists a positive constant* ℓ *such that*

$$f_1(t,x,y) \le \ell\big[1 + f_2(t,x,y)\big] \ \ \textit{for} \ \ a \le t \le b, \ \ x \ge 0, \ \ 0 \le y \le \gamma;$$

(iii) $\beta_1 = 0$, $\beta_2 = 1$*, and*

$$\tau_1(t) < t \ \ \textit{for} \ \ a < t \le b;$$

(iv) $\alpha > 0$, $\beta_1 = 0$, $\beta_2 = 0$*, and there exists a positive constant* ℓ *such that*

$$f_2(t,x,y) \le \ell\big[1 + f_1(t,x,y)\big] \ \ \textit{for} \ \ a \le t \le b, \ \ 0 \le x \le \gamma, \ \ y \ge 0;$$

(v) $\alpha > 0$, $\beta_1 = 1$, $\beta_2 = 0$*, and*

$$\tau_2(t) < 0 \ \ \textit{for} \ \ a < t \le b.$$