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PLURIREGULAR, PLURIGENERALIZED REGULAR EQUATIONS IN CLIFFORD ANALYSIS

The Dirac operator is a powerful tool for the Clifford analysis. It is well-known that the classical equations of mathematical physics, electromagnetic fields, relativistic quantum mechanics and their generalizations are obtained by using the Dirac operator without application of any physical laws. In our paper new higher order equations are obtained by using the Clifford analysis. They have some mathematical beauty and can have important applications.

The pluriregular, plurigeneralized regular and pluri-Beltrami equations

$$\overline{\partial}^{\,m} u(x) = 0, \tag{1}$$

$$P^m u(x) = 0, \quad Pu = \overline{\partial} u + \widetilde{u} h, \tag{2}$$

$$(\overline{\partial} + q\partial)^m u = 0, \ \ |q| \neq 1, \ \ m \geq 2, \ \ x(x_0, \dots, x_n) \tag{3}$$

are considered, which are elliptic in $R(n)$, hyperbolic in $R(n, n-1)$ and pluriparabolic in $R^0(n)$.

If $u(x)$ is the solution of (1) or (2), then it will be the solution of a poly-harmonic or poly-Helmholtz equation in $R(n)$ and poly-wave or poly-Klein-Gordon equation in $R(n, n-1)$, supposing $h = \text{const}$, $\Delta^m u = 0$, $(\Delta - |h|^2)^m u = 0$,

$$\Delta - \frac{\partial^2}{\partial t^2} \ ^m u(x) = 0, \qquad \Delta - |h|^2 - \frac{\partial^2}{\partial t^2} \ ^m u(x) = 0, \quad x_n \equiv t.$$

In a parabolic case we obtain the poly-heat equation

$$\Delta - \frac{\partial}{\partial t} \ ^m u(x) = 0.$$

Moreover, in the Clifford analysis the higher order equations of new type are considered; they are connected with the following equations:

$$\Delta \ \ \Delta - \frac{\partial}{\partial t} \ \ u(x, t) = 0, \ \ x(x_0, \dots, x_{n-1}), \quad x_n \equiv t > 0,$$

$$\Delta \ \ \Delta - \frac{\partial^2}{\partial t^2} \ \ u(x, t) = 0,$$

$$\Delta - \frac{\partial^2}{\partial t^2} \quad \Delta - \frac{\partial}{\partial t} \ \ u(x, \tau, t) = 0, \ \ x(x_0, \dots, x_{n-2}), \quad x_{n-1} \equiv \tau,$$

$$\Delta \ \ \Delta - \frac{\partial^2}{\partial t^2} \quad \Delta - \frac{\partial}{\partial t} \ \ u(x, \tau, t) = 0.$$

The above equations are called harmonic-heat, harmonic-wave, wave-heat and harmonic-wave-heat equations, respectively.

For these equations the boundary, initial and boundary-initial value problems are considered, some of them are solved in quadratures.