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ON A PERIODIC BOUNDARY VALUE PROBLEM FOR THE TWO-DIMENSIONAL LINEAR DIFFERENTIAL SYSTEM

The differential system

$$u_i'(t) = p_{ii}(t)u_i(t) + p_{i,3-i}u_{3-i}(t) + q_i(t) \quad (i = 1, 2) \tag{1}$$

is considered with the periodic boundary conditions

$$u_i(w) = u_i(0) \quad (i = 1, 2), \tag{2}$$

where $p_{ik} : [0, w] \to R$ $(i, k = 1, 2)$ are Lebesgue integrable functions, $w > 0$.

Let

$$p_i(t) = p_{i,3-i}(t) \exp\left(\int_0^t (p_{3-i,3-i}(s) - p_{ii}(s))\, ds\right) \text{ for } 0 \le t \le w,$$

$$\lambda_i = \exp\left(-\int_0^w p_{ii}(s)ds\right) \quad (i = 1, 2), \quad l = \int_0^w |p_1(s)|\, ds \int_0^w |p_2(s)|\, ds,$$

$$\alpha_1 = \min\{1, \lambda_1\lambda_2\}, \quad \alpha_2 = \max\{1, \lambda_1\lambda_2\},$$

$$\beta_1 = \min\{\lambda_1, \lambda_2\}, \quad \beta_2 = \max\{\lambda_1, \lambda_2\}.$$

Then the following statemants are valid.

Theorem 1. *Let*

$$l > 0, \quad l^{-1}(\lambda_1 - 1)(\lambda_2 - 1) \notin]\alpha_1, \alpha_2[$$

and $\sigma \in \{-1, 1\}$ *be such that*

$$\sigma p_{12}(t) \ge 0, \quad \sigma p_{21}(t) \le 0 \text{ for } 0 \le t \le w.$$

Then the problem (1), (2) *has one and only one solution.*

Theorem 2. *Let* $\sigma \in \{-1, 1\}$ *be such that*

$$\sigma p_1(t) \ge 0, \quad \sigma\left(\int_t^w p_2(s)\, ds + \frac{\lambda_2}{\lambda_1}\int_0^t p_2(s)\, ds\right) < 0 \text{ for } 0 \le t \le w$$

and

$$0 < \int_0^w |p_1(s)|\, ds \int_0^w [\sigma p_2(s)]_-\, ds \le 16\,\frac{\beta_1}{\beta_2}\,.$$

Let, moreover,

$$\int_0^w (p_{22}(s) - p_{11}(s))\, ds \int_0^w p_{11}(s)\, ds \ge 0.$$

Then the problem (1), (2) *has one and only one solution.*