N. Lazrieva and T. Toronjadze

**A. Razmadze Mathematical Institute, Georgian Academy of Sciences
Tbilisi, Georgia**

THE ROBBINS–MONRO TYPE SDE. ASYMPTOTIC PROPERTIES OF SOLUTION AND AVERAGING PROCEDURES

We consider the Robbins–Monro family (RM) type SDE introduced in [1], that is,

$$z_t = z_0 + \int_0^t H_s(z_{s-})\,dK_s + \int_0^t M(ds, z_{s-}),$$

where the random field $\{H_t(\omega,u),\, t \geq 0,\, u \in R^1\}$ is such that $(H_t(u))_{t\geq 0}$ is a predictable process for each $u \in R^1$ and

$$H_t(0) = 0,$$
$$uH_t(u) \leq 0 \ \text{ if } \ u \neq 0,$$

for each $t \geq 0$, P-a.s., $K = (K_t)_{t\geq 0}$ is a predictable increasing process, $M(u) = (M(t,u))_{t\geq 0} \in \mathcal{M}^2_{loc}$ for each $u \in R^1$, $\int_0^t M(ds, z_{s-})$ is the stochastic line integral w.r.t. the family $\{M(t,u),\ t \geq 0,\ u \in R^1\}$ along the predictable curve $u = (u_t)_{t\geq 0}$.

Assuming that there exists a unique strong solution $z = (z_t)_{t\geq 0}$ of this equation, the general results concerning the asymptotic behavior of $z = (z_t)_{t\geq 0}$ as $t \to \infty$ are presented. In particular, the rate of convergence $z_t \to 0$ P-a.s. is established. Moreover, it is shown that z admits an asymptotic expansion which enables one to obtain the asymptotic distribution of the randomly normed solution from a martingale limit theorems.

Further, we consider the Polyak weighted averaging procedure

$$\overline{z}_t = \mathcal{E}(-g \circ K) \int_0^t z_s\, d\mathcal{E}_s^{-1}(-g \circ K),$$

where $\mathcal{E}(-g\circ K)$ is the Dolean exponential of the process $g\circ K_t = -\int_0^t g_s\,dK_s$, $g_t \geq 0$.

The asymptotic properties of $\overline{z} = (\overline{z}_t)_{t\geq 0}$ as $t \to \infty$ are investigated.

References

1. N. Lazrieva, T. Sharia, and T. Totonjadze, The Robbins–Monro type SDE, I, Convergance of solutions. *Stochastics and Stochastics Reports* **61**(1997), 67–87.