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ON FOURIER MULTIPLIERS AND THE SOLVABILITY
OF CERTAIN SUPERLINEAR PDE

The main goal of this report is to present two-weighted estimates for the multipliers of Fourier transforms and their application to the multidimensional Ricatti equation.

Let $S(R^n)$ be Schwartz space of rapidly decreasing functions. Denote by $F(\varphi)$ the Fourier transform of $\varphi \in S(R^n)$. Let the operator K defined by the Fourier transform equation

$$F(K\varphi) = mF(\varphi), \quad \varphi \in S.$$

The following statement is true:

Theorem. *Let the function v_j, $j = 1, 2$, satisfy the conditions*
(i) $v_j \in L^1(R^1)$, $j = 1, 2$.
(ii) $\displaystyle\int\limits_{R^1} \frac{1}{v_2(x)|x|^{(1-\alpha)2}}\, dx < \infty.$
Moreover, assume that

$$m(\lambda) = \int\limits_{-\infty}^{\lambda} d\mu,$$

where μ is a finite measure on R^1. Then the following inequality

$$\int\limits_{R^1} |Kf(x)|^2 \frac{1}{w(x)}\, dx \le$$

$$\le \left(\int\limits_{R^1} \frac{1}{v_2(x)|x|^{(1-\alpha)2}}\, dx\right)\left(\int\limits_{R^1} |f(x)|^2 \frac{1}{v_1(x)}\, dx\right)$$

holds, where $w \stackrel{def}{=} v_1 * v_2$.

We intend the application to the solvability problem for equation

$$-\Delta u = |\nabla u|^p + f, \quad 1 < p < \infty,$$

to discuss.