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ON SINGULAR BOUNDARY VALUE PROBLEMS FOR FUNCTIONAL DIFFERENTIAL EQUATIONS

Suppose m and n are natural numbers, $-\infty < a < b < +\infty$, $\alpha : [0, n-1]$, $\beta \in [0, n-1]$,

$$\alpha_i = \frac{\alpha + i - n + |\alpha + i - n|}{2}, \quad \beta_i = \frac{\beta + i - n + |\beta + i - n|}{2} \ \ (i = 1, \dots, n);$$

$\mathbb{R}^m$ is the m-dimensional real Euclidean space with norm $\|\cdot\|_{\mathbb{R}^{\gtrdot}}$;

$C^{n-1}_{\alpha,\beta}(]a, b[\,; \mathbb{R}^{\gtrdot})$ is the Banach space of $(n-1)$–times continuously differentiable vector functions $x :]a, b[\to \mathbb{R}^{\gtrdot}$ having the limits

$$\lim_{t \to a} (t-a)^{\alpha_i} x^{(i-1)}(t), \quad \lim_{t \to b} (b-t)^{\beta_i} x^{(i-1)}(t) \ \ (i = 1, \dots, n),$$

where the norm is defined by the equality

$$\|x\|_{C^{n-1}_{\alpha,\beta}} = \sup\Big\{ \sum_{k=1}^{n} (t-a)^{\alpha_i} (b-t)^{\beta_i} \|x^{(i-1)}(t)\|_{\mathbb{R}^{\gtrdot}} : \ \ a < t < b \Big\};$$

$L_{\alpha,\beta}(]a, b[\,; \mathbb{R}^{\gtrdot})$ is the Banach space of summable with weight $(t-a)^\alpha \times (b-t)^\beta$ vector functions $y :]a, b[\to \mathbb{R}^{\gtrdot}$ with the norm

$$\|y\|_{L_{\alpha,\beta}} = \int_a^b (t-a)^\alpha (b-t)^\beta \|y(t)\|_{\mathbb{R}^{\gtrdot}} \, dt.$$

Sufficient conditions for the solvability and unique solvability are established for the singular boundary value problem

$$x^{(n)}(t) = f(x)(t), \quad h_i(x) = 0 \ \ (i = 1, \dots, n), \tag{1}$$

where $f : C^{n-1}_{\alpha,\beta}(]a, b[\,; \mathbb{R}^{\gtrdot}) \to \mathbb{L}_{\alpha,\beta}(]\partial, [\,; \mathbb{R}^{\gtrdot})$ and $h_i : C^{n-1}_{\alpha,\beta}(]a, b[\,; \mathbb{R}^{\gtrdot}) \to \mathbb{R}^{\gtrdot}$ $(i = 1, \dots, n)$ are continuous operators.