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PLANE ELASTICITY PROBLEMS FOR INFINITE DOMAINS WITH UNKNOWN AND PARTLY UNKNOWN CONTOURS

Mixed problems of the plane elasticity are considered for infinite planes with two holes or with periodically disposed holes of two types. The contours of the holes are sets consisting of unknown curves and given segments. The tangent strain is equal to zero on the whole contour, while the normal strain on the unknown part of the contour and the displacement on the given segments are constant. To find the unknown part of the contour, it is additionally requested the constancy of the tangent normal component on it. Effective methods for solution of these problems are elaborated.

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RESULTS OBTAINED BY V. KUPRADZE IN THE TWO-DIMENSIONAL ELASTICITY THEORY AND THEIR GENERALIZATIONS

Results obtained by V.Kupradze in two-dimensional problems of elastostatics for anisotropic bodies will be discussed. His approach based on the potential theory and the theory of singular (Fredholm) integral equations applied to the basic boundary value problems (BVPs) will be described in detail. For the basic BVPs (the first BVP – the displacement vector is given on the boundary, the second BVP – the stress vector is given on the boundary, the mixed BVP – the displacement vector is given on one part of the boundary and the stress vector is prescribed on the remainig part) V. Kupradze created and justified a very efficient and simple numerical method which will be treated as well.

The above mentioned results have been extended to various directions by many authors. In the talk these generalizations will be presented in detail.

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ASYMPTOTICS FOR CRACK AND MIXED TYPE BOUNDARY VALUE PROBLEMS – THE WIENER–HOPF METHOD

Crack–type boundary value problems (BVPs) deal with 3D anisotropic elastic bodies which have a crack of the form of an arbitrary 2D smooth surface with a smooth boundary; on each side of the crack surface either the DIRICHLET type condition (i.e., the displacement vector) or the NEUMANN type condition (i.e., the stress vector) is given.

We get a mixed type problem if, e.g., two 3D bodies are considered: one is bounded and the other is its exterior complement. On one part of the common smooth surface the bodies have the full contact, while on the complementary part there is a crack.

The method is implemented in four steps.

I. Applying the potential method, the above–mentioned BVPs are reduced to boundary integral (pseudodifferential) equations on an open surface which coinsides, e.g., with the crack surface.

II. Extending the theorem on factorization, the full asymptotic expansion of a solution to PsDE in the vicinity of the boundary of the manifold (L_p–theory) is obtained including an explicit description of the exponents and of the logarithmic terms of the expansion.

III. Applying the established asymptotics of the densities (see Step II), a detailed spatial asymptotics of the solution to BVP is derived. Explicit dependences between plane and spatial coefficients are found.

IV. The obtained information is used to establish dependences between stress and displacement fields on crack faces and crack prolongations, which are extremely important for developing fracture criteria for elastic materials with cracks.

Most of results are shared with O. Chkadua, D. Natroshvili, A. M. Sändig, E. Shargorodsky and W. L. Wendland.

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ON KNESER PROBLEM FOR HIGHER ORDER NONLINEAR ORDINARY DIFFERENTIAL EQUATIONS

Consider the differential equation

$$u^{(n)} = f(t, u, \ldots, u^{(n-1)}), \tag{1}$$

where $n \geq 2$, and $f :]0, +\infty[\times R^n \to R$ is a continuous function such that

$$f(t, 0, \ldots, 0) = 0, \quad (-1)^n f(t, x_1, \ldots, x_n) \geq 0 \ \text{ for } \ t > 0,$$
$$(-1)^{i-1} x_i \geq 0 \ \ (i = 1, \ldots, n). \tag{2}$$

A non-negative solution $u :]0, +\infty[\to R_+$ of the equation (1) is called a *Kneser* solution if it satisfies the inequalities $(-1)^i u^{(i)}(t) \geq 0$ for $t > 0$ $(i = 0, \ldots, n-1)$.

Obviously, each Kneser solution u has a finite or infinite right limit $u(0+)$. If $u(0+) = +\infty$, then the Kneser solution will be called *explosive*.

Theorem 1. *Let, along with* (2)*, the condition*

$$|f(t, x_1, \ldots, x_n)| \leq g(t) h(x_1, t|x_2|, \ldots, t^{n-1}|x_n|)$$
$$\text{for } \ 0 < t < a, \quad (-1)^{i-1} x_i \geq 0 \quad (i = 1, \ldots, n)$$

be fulfilled, where a is a sufficiently small positive number, the functions $g :]0, a] \to R_+$ and $h : R_+^n \to R_+$ are continuous and $\int_0^a \tau^{n-1} g(\tau)\, d\tau < +\infty$. Then for any $c_0 \in R_+$ the equation (1) *has at least one Kneser solution satisfying the initial condition $u(0+) = c_0$.*

Theorem 2. *Let, along with* (2)*, the condition*

$$g_0(t) x_1^\lambda \leq (-1)^n f(t, x_1, \ldots, x_n) \leq g(t) h(x_1, t|x_2|, \ldots, t^{n-1}|x_n|)$$
$$\text{for } \ 0 < t < a, \quad (-1)^{i-1} x_i \geq 0 \quad (i = 1, \ldots, n)$$

be fulfilled, where $a \in]0, +\infty[$, $\lambda \in]1, +\infty[$, the functions $g_0 :]0, a] \to R_+$, $g :]0, a] \to R_+$ and $h : R_+^n \to R_+$ are continuous and

$$0 < \int_0^t \tau^{n-1} g_0(\tau) d\tau \leq \int_0^t \tau^{n-1} g(\tau) d\tau < +\infty \ \text{ for } \ 0 < t \leq a.$$

Then the equation (1) *has at least one non-negative explosive Kneser solution.*

Global two-sided estimates of bounded and explosive Kneser solutions are established.

Acknowledgement

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DIRECT AND INVERSE FLUID-STRUCTURE INTERACTION PROBLEMS (GENERAL ANISOTROPIC CASE)

Three-dimensional mathematical problem of interaction between elastic and scalar oscillation fields are investigated in a general anisotropic case.

In the simplest physical situation, this interaction problem can be described as follows. An acoustic wave propagates in an anisotropic homogeneous fluid domain of infinite extent where a bounded anisotropic elastic structure is immersed. The direct problem is to determine the scattered acoustic pressure in the fluid domain and the three-dimensional elastic field (the displacement field) in the anisotropic structure.

The corresponding mathematical transmission problem is studied by the boundary integral equations method. The non-local approach based on the variational formulation of the problem is considered as well (if the anisotropic elastic body in question is inhomogeneous).

The special analysis is given for the wave scattering problems. The solvability of the non-homogeneous problem with physical transmission data is proved for arbitrary oscillation frequencies.

These results are then applied to prove the uniqueness theorem in the inverse fluid-structure scattering problem.

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ON A CRITERION OF CONVERGENCE OF DIFFERENCE SCHEMES FOR LINEAR BOUNDARY VALUE PROBLEMS FOR SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS

Let an integrable matrix-function $P : [a,b] \to \ ^{n\times n}$, an integrable vector-function $q : [a,b] \to \ ^{n\times n}$, a constant vector $c_0 \in \ ^n$ and a linear continuous operator l acting from the space of all vector-functions $x : [a,b] \to \ ^n$ of bounded variation with the norm $\|x\|_s = \sup\{\|x(t)\| : \ t \in [a,b]\}$ into $\ ^n$ be such that the linear boundary value problem

$$\frac{dx(t)}{dt} = P(t)x(t) + q(t), \tag{1}$$

$$l(x) = c_0 \tag{2}$$

has a unique solution x^0.

Along with the problem (1), (2), consider the difference scheme

$$y(k) - y(k-1) =$$

$$= \frac{1}{m}\big(G_{1m}(k)y(k) + G_{2m}(k)y(k-1) + g_m(k)\big) \quad (k \in \ _m), \tag{1_m}$$

$$L_m(y) = \gamma_m \tag{2_m}$$

$(m = 2, 3 \dots)$, where $\ _m = \{1, \dots, m\}$, $G_{jm} : \ _m \to \ ^{n\times n}$ $(j = 1, 2)$, $g_m : \ _m \to \ ^n$, $\gamma_m \in \ ^n$, and L_m is a linear continuous operator with respect to the norm $\|y\|_m = \max\{\|y(k)\| : \ k = 0, \dots, m\}$.

Necessary and sufficient conditions are given for the sequence of boundary value problems (1_m), (2_m) $(m = 2, 3, \dots)$ to have a unique solution y_m for any sufficiently large m and

$$\lim_{m\to+\infty} \max\Big\{\Big\|y_m(k) - x^0\Big(a + \frac{b-a}{m}\,k\Big)\Big\| : \ \ k = 0, \dots, m\Big\} = 0.$$

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ON A LOGARITHMIC METHOD WITH APPLICATION TO SYMMETRIC HYPERBOLIC SYSTEMS OF NONLINEAR CONSERVATION LAWS

We present a new method for investigating time dependent PDEs. In general features, this method can be formulated as the following 4 steps.

1. Development of a semidiscrete scheme for the time dependent PDE under consideration.
2. Selection of suitable norms.
3. Derivation of apriori estimates in selected norms.
4. Analysis on the basis of the obtained apriori estimates and conclusions for PDE under consideration.

It should be emphasized that the logarithimic norm is a key in performing steps 2 and 3. That is why we call the method logarithmic. Note that it is impossible to select a norm and have the corresponding estimates in the case of arbitrarily constructed semidiscrete schemes. So in order to apply successfully the method, it is necessary to keep in mind possibilities offered by the logarithmic norm when performing step 1. Thus from this viewpoint the approach can be useful for constructing stable numerical schemes as well.

Capabilities of the developed approach is demonstrated by means of its application to symmetric hyperbolic systems of nonlinear conservation laws.

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THEOREMS OF THE UNIQUENESS OF SOLUTIONS OF BOUNDARY VALUE PROBLEMS FOR EVOLUTIONARY EQUATIONS

A survey of the results on the uniqueness of solutions of initial boundary value problems for parabolic equations and systems is given in the present report. The history of this problem takes its origin from the 1930s and is reflected in the works of Tikhonov, Holmgren, Teklund and others. Therein theorems on the uniqueness of solutions of the Cauchy problem for the heat equation in classes of growing functions have been formulated for the first time. Subsequently, the interest to these problems increased after appearance of the well-known papers by Petrovskiĭ.

A large number of works, in particular for second order equations are devoted to investigation of this problem. Very interesting and important results have been obtained by such authors as Oleĭnik, Radkevich, Iosifyan, Kamynin, Khimchenko, Ladyzhenskaya, Smirnova, Landis, Zhitomirskiĭ, Ivasishen, Eidelman, Shishkov, Gagnidze, Solonnikov, Marinov, Kalashnikov, and others. For the last years, in Georgia, the results obtained in this direction by Gagnidze, Gugulashvili and Maisuradze are of great interest.

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ON INTEGRAL REPRESENTATION OF SOLUTION OF THE CAUCHY PROBLEM

For the Cauchy problem $\frac{dx}{dt} = f_t(x), x(t_0) = x_0$, the problem of representation of solution as a field function is considered.

Let $V \subset R^2, M$ be a monoid with compatible algebraic and limiting structures (i.e., the binary operation $*$ is continuous), Σ denote the set of all partitions of the form $\sigma = \{0 = s_0 \leq \xi_1 \leq s_1 \leq \cdots \leq \xi_n \leq s_n = 1\}(s_0 < \cdots < s_n)$ directed in a usual way.

Definition 1. We say that $g \in M$ is the integral of the function $h : V \to M$ from a to b $(a, b \in R)$ and write $g =^{(*)} \int_a^b h(\tau, d\tau)$, if for some $\sigma_0 \in \Sigma$ the directedness

$$\left\{h(a + \xi_n(b-a), \Delta s_n(b-a)) * \cdots * h(a + \xi_1(b-a), \Delta s_1(b-a))\right\}_{\sigma_0 \leq \sigma \in \Sigma}$$

is defined correctly and converges to g.

Definition 2. We say that the field $(t, x) \mapsto f_t(x)$ has Property E, if $(t, x) \mapsto f_t(x)$ maps $[a, b] \times R^r$ into R^r, $-\infty < a, b < \infty, \left\{f_t(x)\right\}_{t \in [a,b]}$ is a continuous family for every $x \in R^r$ and there exists $k \geq 0$ such that

$$\|f_t(x_1) - f_t(x_2)\| \leq k\|x_1 - x_2\|, \forall t \in [a, b], \forall x_1, x_2 \in R^r.$$

$C_{Lip}(R^r)$ denotes the set of Lipschitz's mappings $f : R^r \to R^r$. $C_{Lip}(R^r)$ is provided with compatible limiting and monoid (created by the composition $\circ$ and the identical mapping $I : R^r \to R^r$) structures.

Theorem. *Let $(t, x) \mapsto f_t(x)$ have Property E. Then for every $(t_0, t, x) \in (a, b)^2 \times R^r$ there exists ${}^{(\circ)}\int_{t_0}^t (I + d\tau f_\tau)$, and $\psi(t) = \left({}^{(\circ)}\int_{t_0}^t (I + d\tau f_\tau)\right)(x)$ is a solution of the equation $\frac{dx}{dt} = f_t(x)$ with the initial condition $\psi(t_0) = x_0$.*

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ON BEHAVIOUR OF SOLUTIONS OF A NONLINEAR SINGULAR CAUCHY PROBLEM NEAR THE BOUNDARY OF DOMAINS OF THEIR REGULARITY

We consider the quasilinear mixed type equation

$$(shp)^2 u_{tt} - 2\big[(shp)(shq)ch(p+q)\big]u_{xt} + (shq)^2 u_{xx} = 0, \qquad (A)$$

where p and q denote the first order derivatives of the solution in x and t, respectively. The class of hyperbolic solutions of (A) is defined by the inequality $pq(p+q) \neq 0$. The characteristic roots $\xi_1 = (e^p - 1)(1 - e^{-1})^{-1}$, $\xi_2 = (e^{-p} - 1)(e^q - 1)^{-1}$ at the same time are riemannian invariants for (A). In the half-plane $t \geq 0$, we consider the Cauchy problem with the data

$$u\big|_{t=0} = \tau(x), \quad u_t\big|_{t=0} = \nu(x), \quad x \in [a,b], \qquad (B)$$

where the given functions τ, $\nu \in C^2[a,b]$ guarantee the hyperbolity of (A) on the finite or unbounded support of data. Characteristic curves of (A) under the conditions (B) are represented by the one-parametric families of straight lines $t - \xi_k(\alpha)(x-\alpha) = 0$, $k = 1,2$, $\alpha \in [a,b]$, $\xi_k(\alpha) = \xi_k[\tau'(\alpha), \nu(\alpha)]$. The set of the points of their intersection cover the domain of influence of the data (B). The smoothness of initial perturbations doesn't guarantee not only the regularity of solutions in this domain, but their singlevaluedness as well.

Theorem. *If*

1) $$\tau'(x)\nu(x)[\tau'(x) + \nu(x)] \neq 0,$$

2) $$\xi_2^2(x) + \xi_1(a)[(b-x)\xi_2(x)]' > 0, \quad x \in [a,b],$$

3) $$\xi_1^2(x) + \xi_2(b)[(x-a)\xi_1(x)]' > 0, \quad x \in [a, x_0)$$

and changes the sign at the point $x = x_0$, then the boundary of the regularity domain of the solution of (A), (B) contains the characteristic $t - \xi_2(b)(x - b) = 0$. Each point of the interval $x < x_0$ of this straight line is essentially singular.

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ON A NONLINEAR INTEGRO-DIFFERENTIAL EQUATION

In the cylinder $\Omega \times (0, \infty)$ of the variables $x_1, x_2, \dots, x_n, t$, where $\Omega \subset R^n$ is a bounded domain with the sufficiently smooth boundary $\partial\Omega$, we consider the initial boundary value problem

$$\frac{\partial U}{\partial t} - \sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left[a\left(\int_0^t |\nabla U|^2 d\tau\right)\frac{\partial U}{\partial x_i}\right] = 0, \tag{1}$$

$$U(x,t) = 0, \ \ (x,t) \in \partial\Omega \times (0,\infty), \tag{2}$$

$$U(x,0) = U_0(x), \ \ x \in \Omega. \tag{3}$$

Here $a = a(s)$ and $U_0 = U_0(x)$ are given functions, $\nabla = (\partial/\partial x_1, \dots, \partial/\partial x_n)$.

The equation (1) is a generalization of equations arising in mathematical modelling of the process of penetrating of the electromagnetic field in a substance whose coefficient of electroconductivity depends on the temerature.

The main purpose of our work is to study the asymptotic behaviour as $t \to \infty$ of the solution of the probllem (1)–(3). If $a(s) \geq a_0 = const > 0$ and $U_0 \in L_2(\Omega)$, it is not difficult to obtain the a priori estimate

$$\|U(\cdot,t)\|_{L_2(\Omega)} \leq e^{-ct}\|U_0\|_{L_2(\Omega)}, c = const > 0.$$

The main results are the following theorems.

Theorem 1. *If $a(s) \geq a_0 = const > 0$, $a'(s) \geq 0$, $a''(s) \leq 0$, then the solution of the problem* (1)–(3) *admits the estimates*

$$\int_0^\infty \int_\Omega |\nabla U|^2 dx dt \leq C, \quad \int_0^\infty \left|\frac{d}{dt}\int_\Omega |\nabla U|^2 dx\right| dt \leq C. \tag{4}$$

From (4) we get $\|U(\cdot,t)\|_{W_2^1(\Omega)} \to 0$ as $t \to \infty$.

For the one-dimensional ($n = 1$) analogue the equation (1) with the boundary conditions

$$U(0,t) = 0, \quad U(1,t) = 1, \tag{5}$$

we have

Theorem 2. *If $a(s) = (a + s)^p$, $0 < p \leq 1$, $U_0(0) = 0$, $U_0(1) = 1$, $U_0 \in W_2^2(0,1)$, then the solution of the one-dimensional problem* (1), (3), (5) *admits the estimates*

$$\frac{\partial U}{\partial x} = 1 + O(t^{-1-p}), \quad \frac{\partial U}{\partial t} = O(t^{-1}), \quad t \geq 1.$$

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CONTINUOUS DEPENDENCE AND DIFFERENTIABILITY WITH RESPECT TO INITIAL DATA AND RIGHT-HAND SIDE SOLUTIONS OF DIFFERENTIAL EQUATIONS WITH DEVIATING ARGUMENT

Non-linear differential equations with variable delay and quasi-linear differential equations of neutral type are considered, when at initial moment of time the value of initial function, generaly speaking, do not coincide with the initial value of the trajectory (discontinuity at initial moment). The theorems on continuity of the Cauchy problem solution with respect to initial data and right-hand side are proved. The perturbations of the initial data i.e. of the initial function and initial values (initial moment, initial value of trajectory) are small in the uniform and Euclidean norm, respectively. The pertrurbation of the right-hand side of equation is small in the integral sense. Representation formulas for the differential of solution are obtained, when pertrurbations are small in the Euclidean topology. If the effect of discontinuity at the initial moment influences upon the right-hand side of the equation, then, as distinct from the early obtained formula, representation formulas for the differential contain a new term.

Finally we note that the results obtained in this work play an important role when investigating optimal control problems with deviating argument.

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ON THE SOLVABILITY OF A MULTIDIMENSIONAL BOUNDARY VALUE PROBLEM FOR A HIGHER ORDER NON-STRICTLY HYPERBOLIC EQUATION

In the space R^{n+1} of the variables t and $x = (x_1, \dots, x_n)$, let us consider the fourth order hyperbolic equation

$$^2u + L(\partial)u = F, \tag{1}$$

where $\equiv \frac{\partial^2}{\partial t^2} - \sum\limits_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$, $n \geq 2$, $L(\partial)$ is a linear third order partial differential operator with constant coefficients, $\partial \equiv (\frac{\partial}{\partial t}, \frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_n})$, F is a given function and u is an unknown function.

Denote by $D: t > |x|$ the conic domain bounded by the characteristic cone $S_1 : t = |x|$. Let $P_0(t_0, x_0)$ be an arbitrary point of D and $S_2 : |t-t_0| = -|x - x_0|$ be the light cone of the past with the vertex at P_0. Denote by $D_0 : |x| < t < t_0 - |x - x_0|$ the domain bounded by the characteristic cones S_1 and S_2. Let $S = \partial D_0 \cap S_1$.

Consider the following boundary value problem for the equation (1): find in the domain D_0 a solution $u(t,x)$ of the equation (1) satisfying the boundary conditions

$$u\big|_{S_0} = 0, \quad \frac{\partial u}{\partial n}\Big|_{S_0} = 0, \tag{2}$$

where $\frac{\partial}{\partial n}$ is the derivative along the outer normal to S_0.

Theorem. *Let $L(\xi) = l(\xi)(\xi_0^2 - \xi_1^2 - \dots - \xi_n^2) + L_0(\xi)$, $\xi = (\xi_0, \xi_1, \dots, \xi_n) \in R^{n+1}$, where $l(\xi)$ is an arbitrary linear form and $L_0(\xi)$ is an arbitrary polynomial of second order. Then for any $F \in \overset{\circ}{W}{}_2^2(D_0)$ there exists a unique solution of the problem* (1), (2) *in the space $W_2^4(D_0)$.*

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ON BOUNDED AND TIME-PERIODIC SOLUTIONS OF NONLINEAR WAVE EQUATIONS

In the strip $D_a = R \times [0, a]$, the nonlinear wave equation

$$u_{tt} - u_{xx} + g_1(u)u_t + g(t, x, u) = 0 \tag{1}$$

is considered. It is assumed that $g_1 : R \to R$ and $g : D_a \times R \to R$ are continuously differentiable functions and for any $\rho > 0$

$$\sup \big\{ |g(t, x, z)| : \ \ (t, x) \in \mathcal{D}_a, \ \ |z| \leq \rho \big\} < +\infty.$$

By a solution of the equation (1) we understand a classical solution.

For the equation (1) the following problems on bounded and time-periodic solutions

$$\sup_{(t,x)\in\mathcal{D}_a} \big\{ |u_t(t, x)| + |u_x(t, x)| \big\} < +\infty, \quad u(t, 0) = u(t, a) = 0 \quad \text{for} \quad t \in \quad , \tag{2}$$

$$u(t + \omega, x) = u(t, x), \quad u(t, 0) = u(t, a) = 0 \quad \text{for} \quad (t, x) \in \mathcal{D}_a \tag{3}$$

are studied and sufficient conditions of their solvability are established. In particular, the following theorems are true.

By G_1 denote the function $G_1(z) = \int_0^z g_1(\xi)\, d\xi$.

Theorem 1. *Let there exist positive constants δ, η and $\gamma \in (0, 1)$ such that*

$$|g_1(z)| \geq \delta \quad \textit{for} \quad z \in \quad ,$$

$$|4g(t, x, z) - g_1(z)G_1(z)| \leq \frac{\gamma\delta^2}{1 - \exp(-2a)}|z| + \eta \quad \textit{for} \quad (t, x, z) \in \mathcal{D}_a \times \quad .$$

Then the problem $(1), (2)$ *is solvable. Moreover, if in addition g is ω-periodic in the first argument, then the problem* $(1), (3)$ *is also solvable.*

Theorem 2. *Let $g(t + \omega, x, z) \equiv g(t, x, z)$, $\inf\{|g_1(z)| : z \in \quad \} > 0$ and there exist a continuous function $g_0 : \quad \to \quad$ such that*

$$\liminf_{|z|\to+\infty} g_0(z) \operatorname{sgn}(z) > 0,$$

$$\lim_{|z|\to+\infty} \frac{g(t, x, z) - g_0(z)}{h(z)} = 0 \ \ \textit{uniformly with respect to} \ \ (t, x) \in \mathcal{D}_a,$$

where $h(z) = \min\{|g_0(z)|, |g_0(z)g_1(z)G_1(z)|^{\frac{1}{2}}\}$. Then the problem $(1), (2)$ *has at least one solution.*

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ON A TWO-POINT NON-LINEAR BOUNDARY VALUE PROBLEM

We consider the following two-point boundary value problem

$$u^{(n)} = \sum_{k=1}^{n-1} p_k(t)u^{(k)} + f(t, u, u', \dots, u^{(n-1)}), \tag{1}$$

$$\begin{aligned} u^{(i)}(a) &= \varphi_{1i}(u(a), u'(a), \dots, u^{(n-1)}(a)) \quad (i = 0, \dots, n_0 - 1), \\ u^{(j)}(b) &= \varphi_{1j}(u(a), u'(a), \dots, u^{(n-1)}(a)) \quad (j = 0, \dots, n - n_0 - 1), \end{aligned} \tag{2}$$

where $n \geq 2$, $-\infty < a < b < +\infty$, n_0 is the integer part of $\frac{n}{2}$, each of the functions $p_k : [a, b] \to \mathbf{R}$ for $k \in \{1, \dots, n-1\}$ is continuously differentiable up to order $k-1$ inclusively, the functions $f : [a, b] \times \mathbf{R}^n \to \mathbf{R}$, $\varphi_{1i} : \mathbf{R}^n \to \mathbf{R}$ $(i = 0, \dots, n_0 - 1)$ and $\varphi_{2j} : \mathbf{R}^n \to \mathbf{R}$ $(j = 0, \dots, n - n_0 - 1)$ are continuous.

Moreover, let there exist continuous functions $\alpha_0 : [a, b] \to \mathbf{R}$, $\alpha_i : [a, b] \to \mathbf{R}_+$ $(i = 1, \dots, n_0 - 1)$, $\alpha : [a, b] \to \mathbf{R}_+$ and $h_0 : [a, b] \times \mathbf{R}_+^{n_0} \to \mathbf{R}_+$, and constants $c_i \geq 0$ and $\lambda_i \in [0, 1]$, $i \in \{1, 2\}$, such that the inequalities

$$(-1)^{n-n_0-1} f(t, x_0, x_1, \dots, x_{n-1}) \operatorname{sgn} x_0 \geq -\sum_{i=0}^{n_0-1} \alpha_i(t)|x_i| - \alpha(t),$$

$$\big|f(t, x_0, x_1, \dots, x_{n-1})\big| \leq h\big(t, |x_0|, |x_1|, \dots, |x_{n_0-1}|\big)$$

hold on $[a, b] \times \mathbf{R}^n$, and the conditions

$$\big|\varphi_1(u(a), u'(a), \dots, u^{(n-1)}(a))\big| \leq c_1 \Big(1 + \sum_{k=n_0}^{n-1} |u^{(k)}(a)|\Big)^{-\lambda_1}$$
$$(i = 0, \dots, n_0 - 1),$$

$$\big|\varphi_2(u(b), u'(b), \dots, u^{(n-1)}(b))\big| \leq c_2 \Big(1 + \sum_{k=n-n_0}^{n-1} |u^{(k)}(b)|\Big)^{-\lambda 2}$$
$$(j = 0, \dots, n - n_0 - 1)$$

are fulfilled on $\mathbf{R}^n$.

It is established that under some restrictions on the functions α, h and p_k $(k = 1, \dots, n-1)$ the problem (1), (2) is solvable. The problem of unique solvability of (1), (2) is also investigated.

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SOME INTEGRAL TRANSFORMS IN WEIGHTED SPACES

In the report some integral transforms are considered arising in airfoil equations, two-dimensional scattering of accustic waves by thin obatacles and two-dimensional viscous fluid flow. The main goal of the report is to present author's recent results concerning mapping properties of these transforms in weighted Banach function spaces. Multidimensional analogs of the above mentioned problems will be discussed as well.

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AN INTEGRAL ANALOGUE OF STURM'S COMPARISON THEOREM

Consider the linear ordinary differential equations

$$u'' + p(t)u = 0, \tag{1}$$

$$v'' + q(t)v = 0, \tag{2}$$

where

$$p, q \in L([a,b]; R_+). \tag{3}$$

The following comparison theorem is valid

Theorem 1. *Let the condition* (3) *be fulfilled,* v *be a solution of* (2) *satisfying* $v(t_1) = v'(t_0) = v(t_1) = 0$, $v(t) \neq 0$ *for* $t \in]t_1, t_2[$ $(t_1 < t_0 < t_2)$. *Let, moreover, the condition*

$$\int_{t_0}^{t} [p(s) - q(s)]\, ds\, (t - t_0) \geq 0 \quad \forall t \in [t_1, t_2]$$

be fulfilled. Then for any nontrivial solution u *of* (1) *either* $u(t_1) = u'(t_0) = u(t_2) = 0$ *or there is* $t_* \in]t_1, t_2[$ *such that* $u(t_*) = 0$. *Moreover, if*

$$\int_{t_1}^{t_2} [p(s) - q(s)]\, ds > 0,$$

then the second condition necessarily holds.

Theorem 1 generalizes Sturm's well-known comparison theorem under the conditions (3). It enables one to obtain new sufficient conditions for the oscillation of the equation (1), as well as for the solvability of certain boundary value problems.

Theorem 2. *Let* $p \in L([a,b]; R_+)$ *and*

$$\int_{\frac{a+b}{2}}^{t} \Big[p(s) - \frac{\pi^2}{(b-a)^2}\Big]\, ds \Big(t - \frac{a+b}{2}\Big) \geq 0 \quad \forall t \in [a,b]. \tag{4}$$

Then any solution of (1) *has at least one zero on* $[a,b]$.

Consider the following boundary value problem for the equation (1).

$$u(a) = u(b) = 0. \tag{5}$$

Theorem 3. *Let* $p \in L([a,b]; R_+)$, *the condition* (4) *be fulfilled and* $p(t) \leq \frac{4\pi^2}{(b-a)^2}$, $t \in [a,b]$, $\int_a^b \big[\frac{4\pi^2}{(b-a)^2} - p(t)\big] dt > 0$, $\int_a^b \big[p(t) - \frac{\pi^2}{(b-a)^2}\big] dt > 0$. *Then the problem* (1), (5) *has only the zero solution.*

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ON PROPER SOLUTIONS OF LINEAR ADVANCED DIFFERENTIAL EQUATIONS

We consider the differential equation

$$u^{(n)}(t) = p(t)u(\tau(t)), \tag{1}$$

where $p \in L_{loc}(R_+; R_+)$, $\tau(t) \geq t$ for $t \in R_+$.

Definition. A solution u of (1) is said to be proper if it is defined in a neighbourhood of $+\infty$.

Below sufficient conditions are given for (1) to have a proper solution $u : [t_0, +\infty[\to R$ satisfying

$$u^{(i)}(t) > 0 \quad \text{for} \quad t \geq t_0 \quad (i = 0, \dots, n-1). \tag{2}$$

Theorem 1. *Let* $q \in]0, +\infty[$,

$$\limsup_{t\to+\infty}(\tau(t) - t) = \Delta < +\infty, \quad \inf\left\{\lambda^{-n}e^{\lambda} : \lambda \in]0, +\infty[\right\} \leq q.$$

Let, moreover,

$$\limsup_{t\to+\infty} e^{-\lambda_0 t}\int_0^t e^{\lambda_0 s}p(s)\,ds < \frac{1}{q\lambda_0},$$

where λ_0 *is a root of* $\lambda^{-n}e^{\lambda} = q$. *Then* (1) *has a proper solution satisfying* (2).

Theorem 2. *Let* $q \in]0, +\infty[$,

$$\limsup_{t\to+\infty}\frac{\tau(t)}{t} = \alpha < +\infty, \quad \inf\left\{\frac{\alpha^{\lambda}}{\prod\limits_{i=0}^{n-1}(\lambda - i)} : \lambda \in]n-1, +\infty[\right\} \leq q.$$

Let, moreover,

$$\limsup_{t\to+\infty} t^{n-\lambda_0-1}\int_0^t s^{\lambda_0}p(s)\,ds < \frac{1}{q(\lambda_0 + 1 - n)},$$

where λ_0 *is a root of* $\alpha^{\lambda} = q\prod\limits_{i=0}^{n-1}(\lambda - i)$. *Then* (1) *has a proper solution satisfying* (2).

Remark. If in the hypotheses of Theorems 1 and 2 the additional condition $\int\limits^{+\infty} t^{n-1}p(t)\,dt = +\infty$ is fulfilled, then the solution u satisfies the condition $\lim\limits_{t\to+\infty} u^{(n-1)}(t) = +\infty$.

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A NONLINEAR PARABOLIC EQUATION FOR THE MAXIMAL HEDGING PROBABILITY

We consider a security market model consisting of a stock and a bond, where the stock price process $S = (S_t, t \in [0,T])$ is determined by SDE

$$dS_t = \mu(t, S_t)dt + \sigma(t, S_t)dW_t, \quad S_0 > 0, \quad t \in [0,T],$$

and for simplicity we assume that the bond price is constant. Here W is a standard Wiener process defined on a probability space (Ω, F, P).

Suppose that: the function $\mu : [0,T] \times R \to R$ is measurable, $\sigma : [0,T] \times R \to R$ is continuous and there exist constants $\lambda > 0$, $C > 0$ such that $|\mu(t,x)| \le Cx$, $\lambda x \le |\sigma(t,x)| \le Cx$, $\forall t \le T, x \in R_+$.

Consider an option with the payoff $f(S_T)$, where $f = (f(x), x \in R_+)$ is a nonnegative measurable function. Denote by $\Pi(t,x,f)$ a collection of self-financing strategies (hedges) such that $X_T^\pi = x + \int_t^T \pi_u dS_u \ge f_T$, a.s.. The number $C_t(f) = \inf\{x : \Pi(T,x,f) \ne \emptyset\}$ is called the fair price of the option at the time t. Under the present assumption there exists a unique equivalent (to P) martingale measure P^* for the stock price process, the market model is complete and there is a minimal hedging strategy π^* such that $X_T^{\pi^*} = f(S_T)$ a.s.. Besides,

$$C_t(f) = C(t, S_t), \quad \text{where} \quad C(t,y) = E^*(f(S_T)|S_t = y).$$

If the initial capital is less than the fair price $C(0, S_0)$ of the option, then it is impossible to hedge the financial obligation $f(S_T)$ with probability 1 and the problem is to calculate the maximal probability of hedging and to construct the corresponding hedging strategy. This problem in different settings and cases was studied by Karatzas, Föllmer, Novikov. We consider the maximization problem $P^\pi(x) = P(X_T^\pi = f(S_T)|X_0^\pi = x) \to \max_{\pi\in\Pi}$.

Let $P(x) := \sup_{\pi\in\Pi} P^\pi(x)$ and introduce the value function

$$\tilde V(t,x,y) := \operatorname{ess\,sup}_{\pi\in\Pi(t)} P\{X_T^\pi = f(S_T)|X_t^\pi = x, S_t = y\}.$$

It is evident that $P(x) = \tilde V(0, x, S_0)$.

Theorem. *$\tilde V(t,x,y)$ is the unique solution of the equation*

$$2\sigma^2(t)V_tV_{xx} + 2\mu(t)\sigma^2(t)(V_yV_{xx} - V_xV_{xy}) + \sigma^4(t)V_{yy}V_{xx} - \sigma^2(t)V_{xy} = \mu^2(t)V_x$$

subject to the boundary conditions: (along with the inequality $V_{xx} < 0$)

$$V(T,x,y) = I(x = f(y)), \quad V(t,0,y) = P(f(S_T) = 0), \quad V(t, C(t,y), y) = 1.$$

Moreover, the optimal hedge is given by $\pi_t^* = -\frac{\mu(t)V_x + \sigma^2(t)V_{xy}}{\sigma^2(t)V_{xx}}$.

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ON SOLVABILITY OF TWO-POINT BOUNDARY VALUE PROBLEMS FOR SECOND ORDER NONLINEAR FUNCTIONAL DIFFERENTIAL EQUATIONS

For the functional differential equation

$$u''(t) = p(t)u(\tau(t)) + q(u)(t) \tag{1}$$

we consider the following boundary value problems

$$u(a) = 0, \quad u(b) = 0, \tag{2_1}$$

$$u(a) = 0, \quad u'(b) = 0. \tag{2_2}$$

We imply that $p \in L([a,b])$, $\tau : [a,b] \to [a,b]$ is measurable, $q : C([a,b]) \to L([a,b])$ is a continuous operator and has a summable majorant on each bounded ball from $C([a,b])$. Besides we imply that for any $u \in C([a,b])$ and almost every $t \in [a,b]$ the inequality

$$q(u)(t) \operatorname{sign} u(t) \geq -\delta(t) \tag{3}$$

is satisfied, where $\delta : [a,b] \to [0,+\infty[$ is a summable function.

Theorem 1. *Let $i \in \{1,2\}$, $p(t) \geq 0$ for $a \leq t \leq b$ and*

$$\int_a^b p(s)|\tau(s) - s|ds < 4. \tag{4_i}$$

Then the problem (1), (2_i) *has at least one solution.*

Theorem 2. *Let $i \in \{1,2\}$, $p(t) \geq 0$ for $a \leq t \leq b$ and*

$$\int_a^b p(s)ds < \frac{16}{i(b-a)}. \tag{5_i}$$

Then the problem (1), (2_i) *has at least one solution.*

Theorem 3. *Let $i \in \{1,2\}$ and there exist $\beta \in [0,1]$ such that*

$$\int_a^b |p(s)|(\tau(s) - a)^{\beta}(b - \tau(s))^{\beta(2-i)} ds < \frac{[4^{1-\beta}(b-a)^{\beta}]^{2-i}}{(b-a)^{1-\beta}}. \tag{6_i}$$

Then the problem (1), (2_i) *has at least one solution.*

The condition (5_1), ((6_1)) is optimal, because for any $\varepsilon > 0$, an operator q can be found such that if

$$\int_a^b p(s)ds = \frac{16}{b-a} + \varepsilon \quad \left(\int_a^b |p(s)|ds = \frac{4}{b-a} + \varepsilon \right),$$

then the problem (1), (2_1) has no solution.

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GENERALIZED BELTRAMI EQUATION IN CLIFFORD ANALYSIS

Consider the equation

$$\overline{\partial}u + q_1 \partial u + q_2 \overline{\partial}\,\overline{u} = 0, \tag{1}$$

where $\overline{\partial}$ is a modification of the Dirac operator, ∂ is its conjugate,

$$u(x) = \sum_A u_{{}_A}(x) l_{{}_A}, \quad x = \sum_0^n x_k l_k, \quad q_k(x) = \sum_0^n q_{kj}(x) l_j, \quad k = 1, 2,$$

$l_{{}_A}$ is the basis of the Clifford spaces $R_{(n)}$, $R_{(n,n-1)}$ $(n \geq 1)$, $A = (\alpha_1 \cdots \alpha_k)$, $1 \leq \alpha_1 < \alpha_2 < \cdots < \alpha_k \leq n$, $u_{{}_A}(x)$ and $q_{kj}(x)$ are real functions. In the space of one complex variable $R_{(1)}$, this equation, known as a generalization of the classical Beltrami equation, was considered only as elliptic with the condition $|q_1| + |q_2| < 1$. Using the properties of these spaces, we can prove

Theorem. 1. *In the space $R_{(n)}$ the equation* (1) *will be elliptic if*

$$a) \ \ |q_1| + |q_2| < 1 \quad or \quad b) \ \ \big| |q_1| - |q_2| \big| > 1,$$

will be hyperbolic if

$$c) \ \ \big| |q_1| - |q_2| \big| < 1 \quad and \quad |q_1| + |q_2| > 1,$$

will be parabolic if

$$d) \ \ |q_1| + |q_2| = 1 \quad or \quad \big| |q_1| - |q_2| \big| = 1.$$

Note that the case b) can be reduced to the case a).

2. *In the space $R_{(n,n-1)}$ the equation* (1) *will be hyperbolic with conditions a) and b), will be elliptic with c) and will be parabolic with d).*

We can also prove that the multidimensional Beltrami equation

$$\overline{\partial}u + q\partial u = 0, \quad |q| \neq 1$$

with a constant vector q can be reduced to the equation $\overline{\partial}u = 0$.

That is why in the space $R_{(3,2)}$ it will be related to the Dirac equations of relativity quantum mechanics and the Maxwell equations of electromagnetic fields, and in the space $R_{(2)}$ it will be related to the Moisil-Theodorescu equations. Corresponding initial and boundary value problems can be solved.

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OSCILLATION AND NONOSCILLATION CRITERIA FOR TWO-DIMENSIONAL LINEAR DIFFERENTIAL SYSTEMS WITH DEVIATING ARGUMENTS

Consider the system

$$\begin{aligned} u_1'(t) &= p(t)u_2(\sigma(t)), \\ u_2'(t) &= -q(t)u_1(\tau(t)), \end{aligned} \tag{1}$$

where $p, q \in L_{loc}(\ _+;\ _+)$ and $\tau, \sigma \in C(\ _+;\ _+)$ are such that

$$\sigma(t) \le t \quad \text{for} \quad t \in \ _+,$$

$$\lim_{t\to+\infty} \tau(t) = +\infty, \quad \lim_{t\to+\infty} \sigma(t) = +\infty,$$

$$\int_0^{+\infty} p(t)\,dt = +\infty.$$

A proper solution (u_1, u_2) of the system (1) is said to be *oscillatory* if both u_1 and u_2 have sequences of zeros tending to infinity. If there exists $t_0 \in \ _+$ such that $u_1(t)u_2(t) \ne 0$ for $t \ge t_0$, then (u_1, u_2) is said to be *nonoscillatory.*

Theorem 1. *Let for some* $\lambda \in]0,1[$ *the inequality*

$$\limsup_{t\to+\infty} h^{-\lambda}(t) \int_0^t p(s) \int_s^{+\infty} q(\xi)H(\xi)h^{\lambda}(\xi)\,d\xi\,ds > \frac{(\lambda-2)^2}{4\lambda(1-\lambda)}$$

be fulfilled, where

$$h(t) = \int_0^t p(s)\,ds, \tag{2}$$

$$H(t) = \begin{cases} 1 & \textit{for} \quad \tau(t) \ge t, \\ \dfrac{h(\tau(t))}{h(t)} & \textit{for} \quad \tau(t) < t. \end{cases}$$

Then every proper solution of the system (1) *is oscillatory.*

Theorem 2. *Let for some* $\lambda \in]0,1[$ *,*

$$\int^{+\infty} q(s)h^{\lambda}(\tau(s))\,ds < +\infty$$

and

$$\limsup_{t\to+\infty} h^{-\lambda}(t) \int_0^t p(s) \int_{\sigma(s)}^{+\infty} q(\xi)h^{\lambda}(\tau(\xi))\,d\xi\,ds < 1,$$

where the function h *is defined by* (2)*. Then the system* (1) *has a proper nonoscillatory solution.*

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ON THE SOLVABILITY OF THE WEIGHTED INITIAL VALUE PROBLEM FOR EVOLUTION SINGULAR FUNCTIONAL DIFFERENTIAL EQUATIONS OF HIGHER ORDERS

In the present paper on the basis of the results obtained in [1, 2] optimal, in a certain sence, sufficient conditions for the solvability of the weighted initial value problem

$$u^{(n)}(t) = f(u)(t), \tag{1}$$

$$\lim_{t\to a} \frac{u^{(k)}(t)}{h^{(k)}(t)} = 0 \quad (k = 0, \dots, n-1) \tag{2}$$

are established, where $f \in C^{n-1}([a,b];\ {}^m) \to L_{loc}(]a,b];\ {}^m)$ is a continuous Volterra operator and $h : [a,b] \to [0,+\infty[$ is an $(n-1)$-times continuously differentiable function such that

$$h^{(k)}(a) = 0 \;\; (k = 0, \dots, n-2), \quad h^{(n-1)}(t) > 0 \;\text{ for }\; a < t \le b. \tag{3}$$

A particular case of (1) is the vector differential equation with delay

$$u^{(n)}(t) =$$
$$= f_0\big(t, u(\tau_{10}(t)), \dots, u^{(n-1)}(\tau_{1\,n-1}(t)), \dots, u(\tau_{l0}(t)), \dots, u^{(n-1)}(\tau_{l\,n-1}(t))\big), \tag{4}$$

where $f_0 :]a,b] \times\ {}^{lmn} \to\ {}^m$ satisfies the local Carathéodory conditions, and $\tau_{ik} : [a,b] \to [a,b]$ are measurable functions such that

$$\tau_{ik}(t) \le t \;\text{ for }\; a \le t \le b \quad (i = 1, \dots, l;\;\; k = 0, \dots, n-1).$$

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THE LIMITING AMPLITUDE PRINCIPLE FOR HYPERBOLIC DIFFERENTIAL EQUATIONS WITH PERIODIC COEFFICIENTS

We study the asymptotic behavior as $t \to \infty$ and $x \in [a, b]$ for solutions of the following problem

$$\begin{gathered} u_{tt} - u_{xx} + q(x)u = f(x)e^{-i\omega t}, \quad x \in R^1, \quad t > 0, \\ u\big|_{t=0} = 0, \quad u_t\big|_{t=0} = 0, \quad x \in R^1, \end{gathered} \tag{1}$$

where $q(x)$ is piecewise continuous and periodic with the period 1; $f(x) \in C_0^\infty(R^1)$, $\sup f \subset [0, 1]$; $\omega \geq 0$ is a real number; $b > 0$ is any fixed constant.

Our main result is the following

Theorem. *Let the Hill operator H_0 be positive, $q \neq \mathrm{const}$ and the number ω^2 do not lie on the boundary of the spectrum of the operator H_0. Then the solution of the problem* (1) *has the following form*

$$u(x, t) = -ie^{-i\omega t}\zeta(x, \omega) + \frac{1}{\sqrt{t}}\left[u_1(x, t) + u_2(x, t)\right] + \nu(x, t),$$

where $u_1(x, t)$, $u_2(x, t)$ are known functions, the function $\zeta(x, \omega)$ is determined by the help of the limiting amplitude principle and the function $\nu(x, t)$ admits the estimate

$$|\nu(x, t)| \leq \frac{C(b)}{t}\left\|f\right\|_{L^2}$$

as $t \to \infty$ and $x \in [a, b]$.

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THE MAXIMUM PRINCIPLE FOR QUASILINEAR SYSTEMS WITH A DEVIATING ARGUMENT OF A NEUTRAL TYPE IN THE PRESENCE OF MIXED RESTRICTIONS

We consider the optimal control problem

$$I = h\left(x\left(T\right)\right) \longrightarrow \min \tag{1}$$

under the restrictions

$$\dot{x}\left(t\right) = f\left(x\left(t\right), x\left(t-\Theta\right), \dot{x}\left(t-\Theta\right), u\left(t\right)\right), \tag{2}$$

$$g\left(x\left(t\right), x\left(t-\Theta\right), \dot{x}\left(t-\Theta\right), u\left(t\right)\right) \leq 0, \tag{3}$$

$$x\left(t\right) = \varphi\left(t\right), \qquad -\Theta \leq t \leq 0, \tag{4}$$

where Θ is a fixed positive number, the scalar function h is continuously differentiable, $x(t) \in W_{1,1}^n[-\Theta, T]$, $u(t) \in L_1^r[0, T]$, (see [1], $f = (f^1, \dots, f^n)$, $g = (g^1, \dots, g^m)$. Note that the conditions (2), (3) are fulfilled almost everywhere on $[0, T]$ and the restrictions (3) satisfy the standard conditions of general position (see [2]).

In the problem under consideration the initial function $\varphi(t) \in W_{1,1}^n[-\Theta, 0]$ is a priori fixed and the moment T is assumed to be known.

If the functions h, f^i, $i = \overline{1,n}$ and g^j, $j = \overline{1,m}$, which are continuously differentiable with respect to their arguments, satisfy the conditions of convexity (see [1]) and the vector functions f and g are linear with respect to the control parameters, then we can reduce the problem (1)–(4) to an extremal problem, for which an analog of Pontryagin's maximum principle can be proved using the scheme from [2].

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ANALOGUES OF POISSON'S FORMULA FOR PROBLEMS OF STATIC THERMODIFFUSION

The system of homogeneous equations of elastic thermodiffusion has the form

$$\begin{gathered}\mu\Delta u+(\lambda+\mu)\operatorname{grad}\operatorname{div}u-\gamma_1\operatorname{grad}u_4-\gamma_2\operatorname{grad}u_5=0,\\ \Delta u_j=0,\quad j=4,5,\end{gathered}\tag{1}$$

where $u=(u_1,u_2,u_3)$ is the vector of displacement, u_4 is the deviation of temperature, u_5 is the "chemical potential" of the solid, λ, μ, γ_1, γ_2 are elastic constants.

Let Ω^+ stand for the ball bounded by the sphere $\partial\Omega$ of the radius R, $\Omega^-=R^3\backslash\overline{\Omega}^+$. Consider the following problem: find in the region Ω^+ (Ω^-) a regular solution of the system (1) which satisfies on $\partial\Omega$ the following boundary condition:

$$(I)^{\pm}:\quad [u(z)]^{\pm}=f(z),\quad [u_j(z)]^{\pm}=f_j(z),\quad j=4,5,$$

where $f=(f_1,f_2,f_3)$ and $f_j(z)$, $j=1,2,\dots,5$, are given functions. The solution of the problem $(I)^+$ is given by the formula

$$\begin{aligned}u(x)&=\frac{1}{4\pi R}\int_{\partial\Omega}M^+(x,y)F(y)\,d_yS,\\ u_j(x)&=\frac{1}{4\pi R}\int_{\partial\Omega}\frac{R^2-r^2}{|x-y|^3}\,f_j(y)\,d_yS,\quad j=4,5,\end{aligned}$$

where

$$M^+(x,y)=\|M^+_{ij}(x,y)\|_{3\times5},$$

$$\begin{gathered}M^+_{ij}(x,y)=(1-\delta_{4j})(1-\delta_{5j})\Big[\delta_{ij}\,\frac{R^2-r^2}{|x-y|^3}+\beta(R^2-r^2)\,\frac{\partial^2\Phi(x,y)}{\partial x_i\partial x_j}\Big]-\\ -\frac{\beta(\gamma_1\delta_{4j}+\gamma_2\delta_{5j})}{\lambda+\mu}\,(R^2-r^2)\,\frac{\partial}{\partial x_i}\,\widetilde{\Phi}(x,y),\quad i=1,2,3,\ \ j=1,2,\dots,5,\\ F=(f,f_4,f_5),\\ \Phi(x,y)=\int_0^1\Big[\frac{R^2-r^2t^2}{(R^2-2Rrt\cos\gamma+r^2t^2)^{3/2}}-\frac{1}{R}-\frac{3rt\cos\gamma}{R^2}\Big]\,\frac{dt}{t^{1+\alpha}}\,,\\ \widetilde{\Phi}(x,y)=\int_0^1\frac{R^2-r^2t^2}{(R^2-2Rrt\cos\gamma+r^2t^2)^{3/2}}\,\frac{dt}{t^{\alpha}}\,,\\ \alpha=(\lambda+2\mu)/(\lambda+3\mu)<1,\quad \beta=(\lambda+\mu)/(2(\lambda+3\mu)).\end{gathered}$$

The solution of the problem $(I)^-$ has an analogous form.

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BOUNDARY VALUE PROBLEMS OF GENERALIZED ELECTROELASTICITY

The paper deals with R. A. Toupin's model of generalized electroelasticity. In this model, unlike the classical electroelasticity, the existence of a polarization vector is taken into account.

Boundary value problems of dynamics and pseudo-oscillation for homogeneous unisotropic electroelastic media are studied when on the whole boundary either the displacement vector and the electric potential or the vector of mechanical stresses and boundary charges are given. For these problems, the theorems of existence and uniqueness are proved and properties of solutions are established.

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ASYMPTOTIC DISTRIBUTION OF EIGENELEMENTS OF THE BASIC TWO-DIMENSIONAL BOUNDARY-CONTACT OSCILLATION PROBLEMS OF THE CLASSICAL AND OF THE COUPLE-STRESS ELASTICITY THEORY

The basic two-dimensional boundary-contact oscillation problems of the classical and the couple-stress elasticity theory are considered for a piecewise-homogeneous isotropic media.

Using the Carleman method and the Tauberian theorem (Ikehara), formulas of asymptotic distribution of the eigenvalues and the eigenfunctions are established.

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SOME BOUNDARY-CONTACT PROBLEMS OF THE COUPLE-STRESS ELASTICITY

Boundary-contact problems of the couple-stress elasticity are investigated for anisotropic homogeneous media with contact on a part of the boundary. Here, one medium has the smooth boundary and the other medium has the boundary containing a closed cuspidal edge (the corresponding bihedral angle is equal to 2π).

Theorems on the existence and uniqueness of solutions of boundary-contact problems in Besov and Bessel potential spaces are obtained by using the potential theory and the general theory of pseudodifferential equations on a manifold with a boundary. The smoothness of solutions is studied in closed domains occupied by elastic media. Asymptotic properties of solutions are also studied.

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ON THE CONVERGENCE OF RESIDUALS FOR PROJECTIVE AND PROJECTIVE-ITERATIVE METHODS IN ELLIPTIC PROBLEMS

Approximate projective methods of the solution of elliptic boundary value problems take their origin in the works of Ritz (J.f.d. reine und angewandte Math., 1908, 135, H. 1) and Galerkin (Vestnik inzhenerov 1915, 19).

Elliptic linear problems are discribed by the operator equations

$$Au = f, \quad u \in D(A), \quad f \in H, \tag{1}$$

$$Au + Ku = f, \quad u \in D(A), \quad f \in H, \tag{2}$$

where A is a self-adjoint positive definite operator in the Hilbert space H, $D(K) \supset D(A)$.

The expressions $\delta_n \equiv Au_n - f$, $\delta_n \equiv Au_n + Ku_n - f$ are called residuals, where u_n is the approximate solution.

The first works devoted to the problems of the convergence of residuals to zero, as $n \to \infty$, are due to Mikhlin (DAN SSSR, 1956, 106, No. 3) and Vainikko (DAN SSSR, 1968, 179, No. 5). In their works the coordinate system is formed by the eigen elements of another operator B which is similar to A. The convergence of the residual $Au_h - f$ to zero, as $h \to 0$, in the method of finite elements has been proved by the author under certain conditions (DAN SSSR, 1980, 254, No. 5).

The projective-iterative method has been suggested by Porter and Stirling (IMA J. Numer. Analys., 1993, 13).

The basic results of the author:

i) let the operator $I+KA^{-1}$ be continuously invertible in H, the operator $KA^{-1/2}$ be bounded in H, the exact solution of the problem (2) be $u \in D(A^\alpha)$, $\alpha \geq 1$, and let in the Bubnov–Galerkin method the coordinate system be formed by the eigenelements of an operator B similar to A. Then for the residual of the projective-iterative method when l is fixed and $n \to \infty$, the estimate

$$\|\delta_{n,l}\|_H \equiv \left\|A\widetilde{u}_{n,l} + K\widetilde{u}_{n,l} - f\right\|_H = o\big(\lambda n^{-(\frac{l+1}{2}+\alpha-1)}\big) \tag{3}$$

is valid, where $\widetilde{u}_{n,l}$ is the approximate solution by the projective-iterative method, λ_n is the n-th eigen value of the operator B, as $\lambda_n \to \infty$, $n \to \infty$ and l is the number of projective-iterative cycles, $\widetilde{u}_{n,-1} = u_n$;

ii) under the same conditions for $u \in D(A)$, for the method of finite elements with iterations, the estimate

$$\|\delta_{h,l}\|_H = \left\|A\widetilde{u}_{h,l} + K\widetilde{u}_{h,l} - f\right\|_H = O\big(\|A^{1/2}\Pi^h A^{-1}\|^l \|A^{1/2}\Pi^h u\|\big) \tag{4}$$

is fulfilled, where $\Pi^h \equiv I - \Pi_h$, Π_h is an orthoprojector in H_A.

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PROBLEMS OF STATICS AND DYNAMICS OF THE COUPLE-STRESS ELASTICITY WITH REGARD FOR FRICTION

Static and dynamic problems of the couple-srtress elasticity are investigated for non-homogeneous anisotropic media, when frictions are prescribed on a part of the boundary or on the entire boundary. In particular, in the case of statics we consider the following problem: Suppose that $\Omega \subset \ ^3$ is a bounded region with the boundary Γ, $\Gamma \in C^\infty$, and let $\Gamma = \Gamma_1 \cup \Gamma_2$, $\Gamma_1 \cap \Gamma_2 = \ $.

Problem. Let $\mathcal{G} \in (L^2(\Omega))^6$, $f \in L^\infty(\Gamma_2)$, $|f| \geq m > 0$ almost everywhere on Γ_2, $\Phi \in (H_{1/2}(\Gamma_1))^6$ and $\varphi \in (L^\infty(\Gamma_2))^3$. Find $U \in (H_1(\Omega))^6$ which is a weak solution of the equation

$$\mathcal{M}(x,\partial)U(x) + \mathcal{G}(x) = 0, \quad x \in \Omega,$$

the restriction on Γ_2 of tangential components of force stress are functions of the class $(L^\infty(\Gamma_2))^3$, and

$$U\big|_{\Gamma_1} = \Phi, \quad (\sigma U)_n\big|_{\Gamma_2} = f, \quad \mu U\big|_{\Gamma_2} = \varphi, \quad |(\sigma U)_\top|_{\Gamma_2} < g \Rightarrow u_\top = 0,$$
$$|(\sigma U)_\top|_{\Gamma_2} = g \Rightarrow \exists \lambda \geq 0 : u_\top = -\lambda(\sigma U)_\top,$$

where $\mathcal{M}(x,\partial)$ is the matrix differential operator of statics of the couple-stress elasticity, σU is the vector of force stress, μU is a couple-stress vector; by a_n and $a_\top$ are denoted respectively the normal and the tangential component of the vector $a \in \ ^3$, and $g = \mathcal{F}|(\sigma U)_n|$; $\mathcal{F}$ is a measurable bounded function on Γ_2, $\mathcal{F}(x) \geq \mathcal{F}_0 > 0$.

The cases $\operatorname{mes}\Gamma_1 > 0$ and $\Gamma_1 = \ $ are considered separately. In the latter case, sufficient conditions for the existence of a solution are written out. An analogous problem is investigated in the dynamic case.

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ON THE INVESTIGATION OF NONLINEAR MODELS OF THE THEORY OF MIXTURES

We consider the homogeneous, isotropic elastic mixture of two components

$$\begin{aligned} &\sum_{k=1}^{3} \frac{\partial \sigma^1_{ki}}{\partial x_k}(u^1,u^2) - \pi_i(u^1,u^2) = -\rho_1 F_i(x), \\ &\sum_{k=1}^{3} \frac{\partial \sigma^2_{ki}}{\partial x_k}(u^1,u^2) + \pi_i(u^1,u^2) = -\rho_2 G_i(x), \end{aligned} \qquad x \in \Omega, \tag{1}$$

where $i = \overline{1,3}$, u^1, u^2 are the displacement vectors of the first and second components respectively, $F = (F_1, F_2, F_3)$, $G = (G_1, G_2, G_3)$ are body forces. The components of the partial stresses σ^1_{ki}, σ^2_{ki} and the diffusive force π_i are of the following form

$$\begin{aligned} \sigma^1_{ki} &= 1/2(-\alpha_2 + \lambda_1 e^1_{pp} + \lambda_3 e^2_{qq})\delta_{ki} + \mu_1 e^1_{ki} + \mu_3 e^2_{ki} - \lambda_5 h_{ki}, \\ \sigma^2_{ki} &= 1/2(\alpha_2 + \lambda_4 e^1_{pp} + \lambda_2 e^2_{qq})\delta_{ki} + \mu_3 e^1_{ki} + \mu_2 e^2_{ki} + \lambda_5 h_{ki}, \\ \pi_i &= \frac{\alpha_2 \rho_2}{\rho} \frac{\partial e^1_{pp}}{\partial x_i} + \frac{\alpha_2 \rho_1}{\rho} \frac{\partial e^2_{pp}}{\partial x_i}, \quad \rho = \rho_1 + \rho_2, \end{aligned}$$

where the tensor indices take the values 1, 2, 3 and are subject to the summation convention, α_2, $\lambda_i (i = \overline{1,5})$, $\mu_j (j = \overline{1,3})$ are material constants, ρ_1, ρ_2 are the massdensities of the two solids. We have denoted

$$\begin{aligned} e^s_{ki} = e_{ki}(u^s) &= \frac{1}{2}\Big(\frac{\partial u^s_i}{\partial x_k} + \frac{\partial u^s_k}{\partial x_i} + \frac{\partial u^s_m}{\partial x_k}\frac{\partial u^s_m}{\partial x_i}\Big), \quad s = 1,2, \\ h_{ki} &= \frac{1}{2}\Big(\frac{\partial u^1_i}{\partial x_k} - \frac{\partial u^1_k}{\partial x_i} + \frac{\partial u^2_k}{\partial x_i} - \frac{\partial u^2_i}{\partial x_k} + \frac{\partial u^1_m}{\partial x_k}\frac{\partial u^2_m}{\partial x_i} - \frac{\partial u^1_m}{\partial x_i}\frac{\partial u^2_m}{\partial x_k}\Big). \end{aligned}$$

For the system (1) the following theorem is true.

Theorem. *Let $\Omega \subset$ be a bounded region with the boundary $\Gamma = \partial\Omega$ of the class ^{2}. If*

$$\begin{gathered} \mu_1 > 0, \ \mu_3^2 < \mu_1\mu_2, \ \lambda_5 \le 0, \ \lambda_1 + \frac{2}{3}\mu_1 - \frac{\rho_2}{\rho}\alpha_2 > 0, \\ \Big(\lambda_3 + \frac{2}{3}\mu_3 - \frac{\rho_1}{\rho}\alpha_2\Big)^2 < \Big(\lambda_1 + \frac{2}{3}\mu_1 - \frac{\rho_2}{\rho}\alpha_2\Big)\Big(\lambda_2 + \frac{2}{3}\mu_2 + \frac{\rho_1}{\rho}\alpha_2\Big), \end{gathered}$$

then there exist neighbourhoods F and U of 0 in the spaces $[L^2(\Omega)]^6$ and $V(\Omega) = \{u |\ u \in [H^2(\Omega)]^6,\ u|_\Gamma = 0\}$ respectively, such that for each $f \in F$ the boundary-value problem $Au = f$ has the unique solution $u \in U$, $u = (u^1, u^2)$, where A is the operator, appearing is in the left-hand part of the system (1).

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ON RELATION OF A MATHEMATICAL MODEL OF BARS TO THE THREE-DIMENSIONAL ELASTICITY THEORY

Consider (N_3, N_2) – approximation of a model of bars given in [1, 2]. Further the notation from those works will be used.

Let the width and the thickness of the bar be given by $2h_i(x_1) = h_0^i x_1^{\alpha_i}$, $i = 2, 3$ ($h_0^i = \text{const} > 0$, $\alpha_i = \text{const} \geq 0$), respectively.

Problem 1. Find bounded weighted double moments $\overset{n_3,n_2}{v_j}(x_1) \in C^3(]0,l[) \cap C(]0,l])$, $j = 1,2,3$, $n_i = \overline{0,N_i}$, $i = 2,3$, of the displacement vector components $u_j(x_1, x_2, x_3)$ under the boundary conditions

$$\overset{n_3,n_2}{v_j}(0) = \overset{n_3,n_2}{\varphi_j^0}, \quad j = 1,2,3, \quad n_i = \overline{0,N_i}, \quad i = 2,3, \quad \text{if} \quad (2N_2+1)\alpha_2 + (2N_3+1)\alpha_3 < 1; \tag{1}$$

$$\overset{n_3,n_2}{v_j}(\ell) = \overset{n_3,n_2}{\varphi_j^\ell}, \quad j = 1,2,3, \quad n_i = \overline{0,N_i}, \quad i = 2,3, \tag{2}$$

provided the lateral boundaries ($x_i = \overset{(\pm)}{h}_i(x_1)$, $i = 2,3$, $0 < x_1 < l$) of the bar are arbitrarily loaded. The functions $\overset{n_3,n_2}{\varphi_j^0}$, $\overset{n_3,n_2}{\varphi_j^\ell}$ and the loadings $\overset{(\pm)}{f_j^i}$ are given.

Statement. Problem 1 is correct in the sense of Hadamard.

If we consider the limit as $N_i \to +\infty$, $i = 2,3$, then from (1) we have

$$\alpha_i < \frac{1}{2N_i+1}, \quad \text{and} \quad \lim_{N_i \to +\infty} \alpha_i = 0, \quad i = 2,3.$$

Thus Problem 1 tends to the following three-dimensional model

Problem 2. Find $u_i(x_1, x_2, x_3) \in C^3(v) \cap \begin{cases} C(\bar{V}) \;\; if \; \alpha_i = 0, \; i = 2,3, \\ C(\bar{V} \backslash \{0, x_2, x_3\}) \; if \; \alpha_i > 0 \end{cases}$

subject to the boundary conditions

$$u_j(0, x_2, x_3) = \varphi_j^0(x_2, x_3), \quad j = 1,2,3, \quad \text{if} \quad \alpha_i = 0, \quad i = 2,3;$$

$$u_j(\ell, x_2, x_3) = \varphi_j^\ell(x_2, x_3) \quad j = 1,2,3,$$

$$X_{ij}(x_1, x_2, x_3)\Big|_{x_i = \overset{(\pm)}{h}_i(x_1)} = \overset{(\pm)}{f_j^i}(x_1, x_{5-i}), \quad j = 1,2,3, \quad i = 2,3.$$

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ON A PROBLEM OF BENDING OF A DOUBLE-CONNECTED PLATE BOUNDED BY BROKEN LINES

The problem of bending of a plate bounded by two convex polygons is studied in the case where piecewise constant normal bending moments act at the boundary of the plate, or the angles of rotation of the boundary points are given beforehand. A function mapping conformally the given region onto an annulus is constructed, and the problem of finding the unknown bending of the plate is reduced to the Riemann–Hilbert problem for the annulus. This method allows one to solve the problem efficiently (in quadratures).

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SOLUTION OF SOME BOUNDARY VALUE PROBLEMS FOR NONHOMOGENEOUS ELASTIC BODIES

For an elliptic ring composed of different isotropic materials, the Dirichlet and Neuman problems are considered. The indicated problems are also studied in the case where a composed elliptic ring is strengthened with anisotropic elliptic kernel or the anisotropic infinite environment is strengthened with an isotropic composed elliptic ring.

Some boundary problems for binary isotropic mixtures are also considered.

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ON A SINGULAR INTEGRAL EQUATION WITH DOUBLY PERIODIC KERNEL

In Hydrodynamics, mainly in the theory of hydroturbines, compressors and pumps, the lattice theory is widely used [1]. This theory contains boundary value problems for piecewise analytic functions in the infinite domains cut along the countable number of segments [1]. On the other hand, this problems are closely connected with singular integral equations with doubly periodic kernels [2].

Consider the following integral equation

$$\frac{1}{\pi i}\int_0^a u(t)\cdot K(t,x)dt = if(x),\quad K(t,x)=\zeta(t-x)-\zeta(t+x),\quad x\in[0,a], \tag{1}$$

where a and ω, $0<a<\omega$, are constants, ζ is Weierstrass's "ζ-function" for the periods 2ω and $2ih$, [3]. When t varies on the whole complex plane and $z=x+iy$, $K(t,z)$ is a doubly periodic kernel with respect to t, with periods 2ω and $2ih$, $f(x)\in H[0,a]$, ($f(0)=0$, H is the Hölder class) is a given function and $u(x)$ is an unknown function, satisfying

$$\int_0^a u(t)dt = 0. \tag{2}$$

Theorem. *In the class $H[0,a]$ there exists a unique solution of the equation* (1) *satisfying the condition* (2)*, and this solution is of the form*

$$u(x)=\frac{\sqrt{X(x)}}{\pi}\cdot\int_0^a\frac{f(t)}{\sqrt{X(t)}}\cdot K(t,x)dt,\quad X(z)=C\cdot\frac{\sigma(z-a)\sigma(z+a)}{\sigma^2(z)},$$

where C is a constant, $\sigma(z)$ is Weierstrass's "σ-function" for the periods 2ω and ih, $\sqrt{X}(z)$ is the branch for which $\sqrt{X}(z)\geq 0$, when $X(z)\geq 0$.

Also the following condition must be fulfilled $\int_0^a f(t)/\sqrt{X(t)}\cdot dt=0$.

The theorem is proved by using Villa's formula for the stripe [1, 3] and properties of singular integrals [4].

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AN APPLICATION OF THE LINEAR CONJUGATION PROBLEM TO STUDY OF PROPERTIES OF CONFORMAL MAPPINGS

It is shown that one can establish new properties of conformal mappings of the unit circle onto simply connected domains with non-smooth boundaries using solutions of boundary value problems for analytic functions. In particular, the following assertion is proved. Let D be a simply connected domain bounded by a piecewise smooth curve with the interior angles $\pi\nu_k$ $(0 \le \nu_k \le 2)$ at the angular points t_k $(k = \overline{1,n})$. Let, moreover, $z = z(w)$ be a conformal mapping of the unit circle onto D. Then

$$z'(w) = \prod_{k=1}^{n} (w - c_k)^{\nu_k - 1} z_0(w), \quad t_k = z(c_k), \quad z_0^{\pm 1} \in \bigcap_{p>1} H^p,$$

where H^p is the Hardy class.

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STABILITY OF ORTHOTROPIC SHELLS OF ROTATION CLOSE TO CYLINDRICAL ONES WITH ELASTIC FILLER UNDER TORSION

The stability of orthotropic shells of rotation, which by their form are close to cylindrical ones with an elastic filler, is investigated under the action of torques applied to the shell ends. The filler is assumed to be a solid elastic body contacting (over the outer surface) with the shell. Both the long and the midlength shells are considered. The filler is assumed to be of slipping type. The investigation is performed by solving the equations of stability of the theory of shells and also the equations of equilibrium of the theory of elasticity. The shells of positive and negative Gaussian curvature are considered. For orthotropic shells of rotation, whose generating midsurface is described by a parabolic function, we obtained formulas in dimensionless form and the universal curves of dependence of the critical shearing stress on the elastic parameters of orthotropy, the dimensionless rigidity of the elastic filler and on the dimensionless amplitude of deviation from the cylindrical form. Deviations of order of thickness are considered. It is established that in the isotropic case the critical stresses under small values of rigidity of the elastic filler may essentially differ for the shells under consideration, whereas this difference becomes insignificant as the rigidity of the elastic filler increases. In the orthotropic case, different degrees of the influence of the elastic parameters on the critical loading are revealed for the shells of positive and negative Gaussian curvature. The conditions are obtained under which one may neglect the influence of the boundary conditions on the critical loading.

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NUMERICAL SOLUTION OF ELASTIC PROBLEMS OF THE NON-AXIALLY SYMMETRIC DEFORMATION OF BODIES OF REVOLUTION WITH NON-REGULAR BOUNDARIES

A numerical method of solving 3-D problems involving elastic isotropic bodies of revolution under a non-axially symmetric load is developed. It is the Ritz method in which an expansion in a system of trigonometric functions along a peripheral direction and a two-dimensional finite-element approximation with respect to the coordinates in the meridian plane are used. Singular functions determining the solution near edge lines or conic points are added to the finite-element approximation.

The analysis of a test problem shows that by using the singular function we increase the accuracy of the computation of the displacements near the singularity. However, the addition of the singular functions to the finite-element approximation leads to ill-conditioned stiffness matrix. The bordering techniques of Faddeev and Faddeeva are used to avoid this problem. To calculate the singular function factor with sufficient accuracy the fine subdivision of a region is demanded. This is due to the fact that when the matrix is ill conditioned, the solution depends drastically on the errors of the numerical integration resulting from the calculation of the matrix coefficients. These errors are reduced when the region subdivision is more fine.

The method is used for solving the problem of the deformation of a half-space with a blind hole under the load applied to the lateral surface of the hole. The problem simulates the process of boring a hole in a body which is used to determine the residual stresses in it.

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STRESS CONCENTRATIONS IN THE NON-LINEAR THEORY OF PLATES

In the present report three dimensional problems of the non-linear elasticity theory are reduced to two dimensional problems of non-linear plates by means of I. Vekua method. Then we consider the case of geometrical non-linear plate for approximate $N = 0$, which have the following complex form:

$$\begin{cases} \frac{\partial}{\partial z}[T_{11} - T_{22} + i(T_{12} + T_{21})] + \frac{\partial}{\partial \bar{z}}[T_{11} - T_{22} + i(T_{12} + T_{21})] + F_+ = 0, \\ \frac{\partial T_+}{\partial z} + \frac{\partial \overline{T}_+}{\partial \bar{z}} + F_3 = 0 \quad \left(z = x_1 + ix_2, \ \ \frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x_1} - i\frac{\partial}{\partial x_2} \right) \right), \end{cases}$$

where

$$\begin{cases} T_{11} - T_{22} + i(T_{12} + T_{21}) = (\sigma_{11} - \sigma_{22} + 2i\sigma_{12})\left(1 + \frac{\partial u_+}{\partial z}\right) + \\ \qquad + (\sigma_{11} + \sigma_{22})\frac{\partial u_+}{\partial \bar{z}}, \\ T_{11} + T_{22} + i(T_{12} - T_{21}) = (\sigma_{11} + \sigma_{22})\left(1 + \frac{\partial u_+}{\partial z}\right) + \\ \qquad + (\sigma_{11} - \sigma_{22} - 2i\sigma_{12})\frac{\partial u_+}{\partial \bar{z}}, \\ T_+ = T_{13} + iT_{23} = \sigma_+ + (\sigma_{11} - \sigma_{22} + 2i\sigma_{12})\frac{\partial u_3}{\partial z} + (\sigma_{11} + \sigma_{22})\frac{\partial u_3}{\partial \bar{z}}. \end{cases}$$

Here

$$\begin{cases} \sigma_{11} - \sigma_{22} + 2i\sigma_{12} = 4\mu\left(\frac{\partial u_+}{\partial \bar{z}} + \frac{\partial u_+}{\partial \bar{z}}\frac{\partial \bar{u}_+}{\partial \bar{z}}\right), \\ \sigma_{11} + \sigma_{22} = 2(\lambda + \mu)\left(\theta + \frac{\partial u_+}{\partial z}\frac{\partial \bar{u}_+}{\partial \bar{z}} + \frac{\partial \bar{u}_+}{\partial z}\frac{\partial u_+}{\partial \bar{z}}\right), \\ \sigma_+ = 2\mu\frac{\partial u_3}{\partial \bar{z}} \quad \left(\theta = \frac{\partial u_+}{\partial z} + \frac{\partial \bar{u}_+}{\partial \bar{z}} \ \ u_+ = u_1 + iu_2\right). \end{cases}$$

The boundary condition can be written as follows:

$$\begin{cases} T_{(ll)} + iT_{(ls)} = \frac{1}{2}\Big\{T_{11} + T_{22} + i(T_{12} - T_{21}) + \\ \qquad + [T_{11} - T_{22} + i(T_{12} + T_{21})]\left(\frac{d\bar{z}}{ds}\right)^2\Big\}, \\ T_{(ls)} = -Im\left(T_+\frac{d\bar{z}}{ds}\right). \end{cases}$$

We solve problems of concentration of stresses around holes by means of the small parameter method.

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THE REVERSE AND HALF-REVERSE PROBLEMS OF ELASTICITY FOR PLANES WEAKENED BY SYMMETRICALLY POSED HOLES

The paper deals with problems of a certain class for the elastic plane weakened by symmetrically posed holes with unknown and partly unknown contours. The tangent strain is zero on the contour of the hole while the normal strain on the unknown part of the hole and the normal displacement on the known part are constant. It is requested to find the unknown part of the contour and the distribution of the strains for the case where the tangent normal strain is constant.

Using methods of the theory of analytical functions, a method of solving is given for the problems mentioned above. Solutions of some concrete problems are given and the sought for contours are constructed.

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ON SOME PROPERTIES OF TIMOSHENKO'S SYSTEM

The paper deals with the problem of solvability of the nonlinear system of the plates theory

$$\frac{\partial N_i}{\partial x_i} + \frac{\partial T}{\partial x_j} + p_i = 0,$$
$$\frac{\partial Q_1}{\partial x_1} + \frac{\partial Q_2}{\partial x_2} + \frac{\partial}{\partial x_1}\Big(N_1\frac{\partial w}{\partial x_1} + T\frac{\partial w}{\partial x_2}\Big) + \frac{\partial}{\partial x_2}\Big(T\frac{\partial w}{\partial x_1} + N_2\frac{\partial w}{\partial x_2}\Big) + q = 0,$$
$$\frac{\partial M_i}{\partial x_i} + \frac{\partial H}{\partial x_j} - Q_i = 0.$$

Here

$$N_i = \frac{Eh}{1-\nu^2}\Big\{\frac{\partial u_i}{\partial x_i} + \frac{1}{2}\Big(\frac{\partial w}{\partial x_i}\Big)^2 + \nu\Big[\frac{\partial u_j}{\partial x_j} + \frac{1}{2}\Big(\frac{\partial w}{\partial x_j}\Big)^2\Big]\Big\},$$
$$T = \frac{Eh}{2(1+\nu)}\Big(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} + \frac{\partial w}{\partial x_1}\frac{\partial w}{\partial x_2}\Big), \quad Q_i = k_0^2\frac{Eh}{2(1+\nu)}\Big(\frac{\partial w}{\partial x_i} + \psi_i\Big),$$
$$M_i = \frac{Eh^3}{12(1-\nu^2)}\Big(\frac{\partial \psi_i}{\partial x_i} + \nu\frac{\partial \psi_j}{\partial x_j}\Big),$$
$$H = \frac{Eh^3}{24(1+\nu)}\Big(\frac{\partial \psi_1}{\partial x_2} + \nu\frac{\partial \psi_2}{\partial x_1}\Big),$$

$u_i(x)$, $w(x)$, $\psi_i(x)$ are unknown functions satisfying certain boundary conditions, $p_i(x)$, $q(x)$ are given functions, $x=(x_1,x_2)$, $i,j=1,2$, $i\neq j$, E, h, ν, k_0 are known positive parametrs and $\nu<1$.

The results are also given for Timoshenko's one-dimensional model.

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ON GENERALIZED RIEMANN PROBLEM

Let D^+ and D^- be inner and outer domains with the boundary Γ, where Γ is a simple, closed Carleson curve such that at each point $t \in \Gamma$ it is homeomorphic to some logarithmic star. $L_p(\Gamma, \omega)$, $1 < p < \infty$, is the weighted Lebesgue space with the Muckenhoupt weight.

Generalized Riemann problem: look for an analytic function $\Phi(z)$ which can be represented as a Caushy integral with a density from $L_p(\Gamma, \omega)$ and whose boundary values $\Phi^+(t)$ from D^+ and $\Phi^-(t)$ from D^- coupled by the following condition:

$$\Phi^+(t) = a(t)\Phi^-(t) + b(t)\overline{\Phi^-(t)} + f(t), \quad t \in \Gamma,$$

where $a,\, b \in C(\Gamma)$, $f \in L_p(\Gamma, \omega)$.

For each $t \in \Gamma$ local parameters $0 < \alpha(t) < 2\pi$, $-\infty < \delta(t) < +\infty$, $0 < \eta_1(t) \leq \eta_2(t) < 1$ are defined. In particular: if Γ is a piecewise smooth curve, then $\alpha(t)$ is the angular measure at the point t and $\delta(t) = 0$; if ω is a Khvedelidze weight, $\omega(\tau) = |\tau - t|^\beta$, $\tau \in \Gamma$, $-\frac{1}{p} < \beta < 1 - \frac{1}{p}$, then $\eta_1(t) = \eta_2(t) = \beta + \frac{1}{p}$.

Theorem. *General Riemann problem is Fredholm if and only if*

$$|a^2(t)| - |b^2(t)|\Big(\frac{sh[(\pi - \alpha(t))(\xi + i(-\delta(t)\xi + \eta))]}{sh\pi(\xi + i(-\delta(t)\xi + \eta))}\Big)^2 \neq 0$$

for all $t \in \Gamma$, $-\infty \leq \xi \leq +\infty$, $\eta_1(t) \leq \eta \leq \eta_2(t)$.

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ON A BOUNDARY PROBLEM FOR ELLIPTIC SYSTEMS AND ITS APPLICATION IN THE MAPPING PRACTICE

A uniformly elliptic first order linear system with coefficients of certain smoothness is considered in a finite n-connected $(n \geq 2)$ domain bounded with piecewise analytic contours. The so-called mixed boundary problem is solved for such systems.

It is shown that the solution of the above problem makes it possible to construct related quasiconformal mappings. For practical reasons the case is especially considered where the domain is either doubly or triply connected and is mapped to concrete canonical domains.

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ON APPROXIMATE SOLUTION OF SOME SINGULAR INTEGRAL EQUATIONS

As is known, one of the convenient methods of investigation of boundary value problems of mathematical physics and elasticity theory is the application of Cauchy type integrals, which reduces the original problems to certain boundary integral equations. For their solution the schemes of approximation of singular integrals prove to be quite effective.

In the present report questions of numerical solution of a number of concrete boundary value problems are considered, based on representation of the solution in the potential form $\operatorname{Re}\Phi(z)$, $\Phi(z) = \frac{1}{\pi}\int\limits_L \frac{\varphi(t)}{t-z}dt$ (where z belongs to the considered domain in the complex plane, and φ is an unknown real function).

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THE BENDING PROBLEM FOR THE FINITE PLATE WITH THE ELASTIC INCLUSION

The problem of the isotropic, circle formed plate's bending is considered when it contains the thin elastic inclusion of variable rigidity and the plate's contour is bound rigidly.

The existence of the inclusion implies the skip of the across force: $< N_y >= \mu(x)$, $|x| < a$, while plate's bending, the angle of rotation and the moment of bending are continuous when going over the segment of the inclusion. The equilibrium equation of the inclusion has the following form:

$$\frac{d^2}{dx^2} D_0(x) \frac{d^2}{dx^2} \omega_0(x) = -\mu(x), \quad |x| < a, \tag{1}$$
$$\left[D_0(x)\omega_0''(x)\right]\Big|_{x=\pm a} = 0, \quad \left[D_0(x)\omega_0''(x)\right]'\Big|_{x=\pm a} = 0,$$

where $D_0(x)$ is the inclusion rigidity and $\omega_0(x)$ is its bending.

Using the methods of the theory of analitical functions and taking into account the contact conditions, we get the following integro-differential equation with respect to the sought for function $\mu(x)$:

$$\nu(x) - \frac{D_0(x)}{8\pi D} \int_{-a}^{a} \frac{v'(t)\,dt}{t-x} + \frac{D_0(x)}{8\pi D} \int_{-a}^{a} R(x,t) v'(t)\,dt = \\ = D_0(x) f(x), \quad |x| < a, \tag{2}$$

where $v''(x) \equiv \mu(x)$, $v(\pm a) = 0$, $v'(\pm a) = 0$, D is the cilindrical rigidity of the plate, $R(x,t)$ is the regular kernel in the square $-a \leq (x,t) \leq a$. The characteristic part of the equation (2) under certain conditions can be reversed and reduced a Fredholm's integral equation of the second kind.

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HILBERT–SHMIDT TYPE THEOREMS IN THE LINEAR MULTIGROUP TRANSPORT THEORY

As it is well known, the Fourier transformation of the solution of the linear multigroup transport equations satisfy the following system of inhomogenous integral equations

$$(\nu I - \mu l)\widetilde{\phi} = \nu \int_{-1}^{+1} g\widetilde{\phi} d\mu' + f, \quad \mu \in (-1;+1), \tag{A}$$

where ν is a parameter, l is a diagonal matrix, $g(\mu;\mu') \in H$ (the Hölder class) and $f(\mu) \in H^*$ (the Muskhelishvili class) are square and column matrices respectively.

Using a result from [1], we study the possibility of representation of the solution of (A) by regular and singular eigenfunctions of the homogeneous system (characteristic system) corresponding to (A). Namely, the following theorem is true:

Theorem. *If $\nu \notin \{\nu_i\} \cup [-1;+1]$, then the unique solution of (A) can be represented in the form*

$$\widetilde{\phi}_\nu(\mu) = \sum_i \frac{\nu_i}{\nu - \nu_i} \phi_{\nu_i}(\mu) \frac{1}{N(\nu_i)} \int_{-1}^{+1} \phi_{\nu_i}(\mu') f(\mu') d\mu' +$$
$$+ \int_{-1}^{+1} \frac{t}{\nu - t} \phi_t(\mu) N^{-1}(t) \int_{-1}^{+1} \phi_t(\mu') f(\mu') d\mu' dt,$$

where $\phi^*_{\nu_i}$ are the regular eigenfunctions and ϕ^*_ν are the singular ones of the adjoint characteristic system, the superscript T means transpose, $N(\nu_i)$ and $N(\nu)$ are the normalisation coefficient and matrix, respectively.

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ON SOLUTIONS OF TWO-DIMENSIONAL BOUNDARY VALUE PROBLEMS OF STABLE OSCILLATIONS OF THE THEORY OF ELASTIC MIXTURES

In the work the basic boundary value problems of the linear theory of two isotropic similar mixtures are investigated. The equations are of the form

$$a_1 \Delta u' + b_1 \operatorname{grad} \operatorname{div} u' + c \Delta u'' + d \operatorname{grad} \operatorname{div} u'' + \rho_1 \sigma^2 u' = \Psi',$$
$$c \Delta u' + d \operatorname{grad} \operatorname{div} u' + a_2 \Delta u'' + b_2 \operatorname{grad} \operatorname{div} u'' + \rho_2 \sigma^2 u' = \Psi'',$$

where Δ is the Laplace operator, ρ_1 and ρ_2 are the partial densities, $\Psi' = \{\Psi'_1, \Psi'_2\}$ and $\Psi'' = \{\Psi''_1, \Psi''_2\}$ are the mass forces, $u' = \{u'_1, u'_2\}$ and $u' = \{u''_1, u''_2\}$ are partial displacements depending on the variables x_1 and x_2, σ^2 is the oscillation frequency, a_1, a_2, b_1, b_2, c, d are constants which charaterise mechanical properties of the elastic mixture in question. They satisfy certain conditions.

With the aid of the potential method and the theory of singular integral equations, existence and uniqueness theorems are proved for basic external boundary value problems of stable oscillations in the case where either the displacement or the generalized stress vector is known on the boundary.

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THE CONNECTION BETWEEN PLANE WAVES AND EIGENFREQUENCIES IN THE THEORY OF TWO-COMPONENT ELASTIC MIXTURES

Existence theorems of eigenfrequencies of interior homogeneous boundary value problems of steady oscillations in the theory of two-component elastic mixtures are proved. Some basic properties of wave numbers are treated. The connection between plane waves and eigenfrequencies is established.

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ON REFINED THEORIES OF ELASTIC SHELLS

On the STAMM-94 [1] we formulated some unsolved mathematical problems of nonlinear solid mechanics. Below we continue to adduce some new results which substantiate the possibility of research and solving of the corresponding mathematical problems.

1. A class of refined theories in Reissner – von Karman sense is constructed with control parameters for elastic shallow shells and geometrical and physical simplifying assumptions. Same problems will be discussed also for non-shallow cases.

2. Numerical processes and characterizing data of a design of some problems for refined theories of elastic plates with boundary layers will be adduced.

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ON THE BIFURCATION PROBLEM OF A STRAIGHT ROD UNDER TORSION

We investigate the problem on bifurcation of equalibrium forms of the axis of a rod which is immovably fastened at one end, has a free cylindrical fixing at another one and is loaded by the torsion moment H.

The problem is described by a boundary-value problem for a system of three non-linear ordinary differential equations dependent on five real parameters, H being the main one. The corresponding branching equation is two-dimensional with a small real parameter.

It turns out that bifurcation points not always appear; we establish conditions when this is the case.

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