Completions of Heyting algebras and bi-relational semantics for IPC

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In this talk, I will review some recent results regarding bi-topological representations for distributive lattices and Heyting algebras. In particular, I will define the notion of a refined regular open set in bi-topological spaces where one topology is finer than the other one, and show how to constructively embed any Heyting algebra into the refined regular open sets of some bi-topological space. I will also present related ideas regarding the representation of well-known completions of Heyting algebras, including MacNeille completions and canonical extensions, as refined regular opens of some bi-topological space. Finally, I will connect these results to bi-relational semantics for **IPC**, also known as FM semantics. In particular, I will show that bi-relational semantics is as general as a semantics for **IPC** in terms of complete Heyting algebras, and give some preliminary results regarding intermediate logics in a bi-relational setting.