One Modal Logic to Rule Them All?

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This is a joint work with Wesley H. Holliday (UC Berkeley) [2].

We introduce an extension of the modal language with what we call the *global quantificational modality* $[\forall p]$. In essence, this modality combines the propositional quantifier $\forall p$ with the global modality A: $[\forall p]$ plays the same role as the compound modality $\forall p \mathsf{A}$. Clearly, we can switch between $\forall$ and $\exists$ and between A and E, thus obtaining other global quantificational modalities—$\langle \exists p \rangle$, $[\exists p]$ and $\langle \forall p \rangle$—but in classical logic they are all definable using $[\forall p]$.

Unlike the propositional quantifier $\forall p$ by itself, the global quantificational modality $[\forall p]$ can be straightforwardly interpreted in any Boolean Algebra Expansion (BAE); for example, it does not requite any form of lattice-completeness. We present a Hilbert-style calculus GQM for this language and prove that it is complete with respect to the intended algebraic semantics by exhibiting an equivalence between a suitable global consequence relation of GQM and the first-order theory of discriminator BAEs. Our formalism, however, is more succinct.

On the other hand, theorems of GQM valid over *lattice-complete* BAEs cannot be axiomatized. In fact, our formalism becomes as bad as the full second-order propositional modal logic with A. This is shown by generalizing normal form results obtained by ten Cate [1].

The logic GQM enables a conceptual shift, as what have traditionally been called different "modal logics" now become $[\forall p]$-universal theories over the base logic GQM: instead of defining a new logic with an axiom schema such as $\Box\varphi \to \Box\Box\varphi$, one reasons in GQM about what follows from the globally quantified formula $[\forall p](\Box p \to \Box\Box p)$.

If time permits, I will also discuss briefly a Coq formalization of our syntactic deductions developed at FAU by Michael Sammler under my supervision.

References

[1] ten Cate, B., *Expressivity of second order propositional modal logic*, Journal of Philosophical Logic **35** (2006), pp. 209–223.

[2] Holliday, W. H. and T. Litak, *One modal logic to rule them all? (extended technical report)* (2018), UC Berkeley Working Paper in Logic and the Methodology of Science. URL https://escholarship.org/uc/item/07v9360j