INFINITARY LOGIC AND BASICALLY DISCONNECTED SPACES

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Riesz Spaces (vector lattices) are lattice-ordered linear spaces over the field of real numbers $\mathbb{R}$. They have had a predominant rôle in the development of functional analysis over ordered structures, due to the simple remark that most of the spaces of functions one can think of are indeed Riesz Spaces. Less known is the rôle that such spaces play in logic: one can consider MV-algebras – the variety of algebras that model Łukasiewicz logic – and endow them with a scalar multiplication, where scalars are elements of the standard MV-algebra $[0, 1]$. These MV-algebras with scalar multiplication form a variety and they are known in literature with the name of Riesz MV-algebras. Moreover, Riesz MV-algebras are categorical equivalent with Riesz Spaces with a strong unit, when both categories are endowed with suitable morphisms. Henceforth, vector lattices and Łukasiewicz logic are closely related.

In this talk we will exploit the connection between Riesz Spaces and MV-algebras to deepen the link between functional analysis and Łukasiewicz logic. To so do, we will provide a logical characterization of uniform convergence, we will characterize different norm-completions of the free Riesz MV-algebra (which is isomorphic to the Lindenbaum-Tarski algebra of the logic modeled by Riesz MV-algebra) and we will define an infinitary logical systems, namely $\mathcal{IRL}$, whose models are intervals spaces of the type $C(X)$, with X a basically-disconnected compact Hausdorff space. We'll further discuss completeness of $\mathcal{IRL}$ with respect to σ-complete Riesz MV-algebras and characterize the Lindenbaum-Tarski algebra of it by means of Borel-measurable functions and Baire functions.

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