Neighbourhood and topological completeness for quantified pretransitive modal logics

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Given a modal logic L the minimal predicate modal logic containing L is QL.

Let L be a modal logic axiomatized by closed formulas and formulas of the form $\Box p \to \Box^n p$ $n \geq 0$ (such logics we call pretransitive Horn logics). Then logic QL is complete with respect to Kripke frames with expanding domains. It is known that these logics are complete with respect to Kripke frames with expanding domains. Logic $S4$ is of this form. It was proved by Rasiowa & Sikorski (1963) that $QS4$ is compete with respect to topological spaces with constant domain. P. Kremer in 2014 proves that $QS4$ is compete with respect to rational numbers with constant domain. Neighbourhood frames is a generalization of topological spaces. So we prove a generalisation of Rasiowa & Sikorski result: for a pretrasitive Horn logics L logic QL is complete with respect to neighbourhood frames with constant domain. As a consequence from our construction we get the Kremer result and a new result that logic $QD4$ is complete with respect to rational numbers with the derivational semantics and the constant domain.