On the variety of $L_P G$ -algebras

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An algebra $(A, \otimes, \oplus, *, \wedge, \vee, \rightarrow, 0, 1)$, is called *$L_P G$-algebra* if $(A, \otimes, \oplus, *, 0, 1)$ is L_P-algebra (i. e. an algebra from the variety generated by perfect *MV*-algebras [2]) and $(A, \wedge, \vee, \rightarrow, 0, 1)$ is a Gödel algebra (i. e. Heyting algebra satisfying the identity $(x \rightarrow y) \vee (y \rightarrow x) = 1$).

Example. The algebra $(C, \otimes, \oplus, *, \wedge, \vee, \rightarrow, 0, 1)$, where $(C, \otimes, \oplus, *, 0, 1)$ is Chang *MV*-algebra [1], is $L_P G$ -algebra.

Let $L_P G$ be the logic corresponding to the variety $\mathbf{L_P G}$ of $L_P G$ -algebras.

- *A lattice of congruences of an $L_P G$ -algebra* $(A, \otimes, \oplus, *, \wedge, \vee, \rightarrow, 0, 1)$ *is isomorphic to a lattice of congruences of the MV-algebra* $(A, \otimes, \oplus, *, 0, 1)$..
- *The variety* $\mathbf{L_P G}$ of *$L_P G$ -algebras is generated by the algebras* $(C, \otimes, \oplus, *, \wedge, \vee, \rightarrow, 0, 1)$.
- *Any $L_P G$ -algebra is bi-Heyting algebra*.
- *The set of theorems of the logic $L_P G$ is recursively enumerable.*
- *A description of finitely generated free $L_P G$ -algebras and characterization of finitely generated projective $L_P G$ -algebras are given.*

References

[1] C. C. Chang, *Algebraic Analysis of Many-Valued Logics*, Trans. Amer. Math. Soc., 88(1958), 467-490.
[2] A. Di Nola, A. Lettieri, *Perfect MV -algebras are Categorically Equivalent to Abelian ℓ-Groups*, Studia Logica, 88(1994), 467-490.