NEW RESEARCH DIRECTIONS IN THE GEOMETRY OF INTUITIONISTIC LOGIC

VINCENZO MARRA

Tarski's ground-breaking 1938 spatial completeness theorem for inutitionistic logic, in one formulation, reads: For any formula in intuitionistic logic that is not provable there is a valuation into the Heyting algebra of open subsets of the real line that provides a countermodel to the formula. In recent joint work with N. Bezhanishvili, D. McNeill, and A. Pedrini, we proved a geometric version of Tarski's theorem. We replace topological spaces by compact polyhedra, and frames of open sets by (not necessarily complete) Heyting algebras of open subpolyhedra. Our result is formally incomparable to Tarski's; geometrically, however, it is far more satisfactory. For instance, a key ingredient of our theorem is that the dimension theory of polyhedra admits an equational reformulation based on the bounded-depth formulæ. In the first part of the talk I give an account of this theorem.

The second part of the talk deals with even more recent work in progress, in which I again study the intermediate logic of simplicial complexes (=triangulations of polyhedra), with the deceptively minor difference that faces are now ordered by *reverse* inclusion. The results, though not quite as conclusive as in the other case yet, potentially hold great interest from several points of view — for intermediate logics, for combinatorial topology, and for categorial logic (through the pre-sheaf topos of simplicial sets).

E-mail address: vincenzo.marra@unimi.it

Dipartimento di Matematica "Federigo Enriques", Università degli Studi di Milano, Italy