Duality for sheaf representations of distributive-lattice-ordered algebras

Sam van Gool

In this talk on joint work with Mai Gehrke, we will discuss the relationship between Stone duality and sheaf representations of ordered algebras, whose order is that of a distributive lattice.

Sheaves were classically used in algebraic geometry to study commutative rings and modules over them. In universal algebra, it is well-known that a sheaf representation over a Boolean base space corresponds to a weak Boolean product decomposition. In case the represented algebra in question is a distributive lattice D with dual space X, such a weak Boolean product decomposition over an indexing space I corresponds to a so-called "Boolean sum decomposition" of the space X over the index space I. Recent developments in the theory of MV-algebras prompted us to study sheaf representations and duality in more detail. In particular, we aim to generalize the above correspondence to indexing spaces I which no longer have a canonical basis, namely stably compact spaces. Specifically, we study the following question:

Question. If A is a distributive-lattice-ordered algebra, how can sheaf representations of A over a stably compact space I be characterized dually?

We will report our current state of knowledge on this general question, and discuss the application of our work to MV-algebras. We will also briefly point to other algebraic structures that naturally arise in logic, to which similar methods could possibly be applied.