Duality for sheaf representations of distributive-lattice-ordered algebras
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In this talk on joint work with Mai Gehrke, we will discuss the relationship between Stone duality and sheaf representations of ordered algebras, whose order is that of a distributive lattice.

Sheaves were classically used in algebraic geometry to study commutative rings and modules over them. In universal algebra, it is well-known that a sheaf representation over a Boolean base space corresponds to a weak Boolean product decomposition. In case the represented algebra in question is a distributive lattice $D$ with dual space $X$, such a weak Boolean product decomposition over an indexing space $I$ corresponds to a so-called “Boolean sum decomposition” of the space $X$ over the index space $I$. Recent developments in the theory of MV-algebras prompted us to study sheaf representations and duality in more detail. In particular, we aim to generalize the above correspondence to indexing spaces $I$ which no longer have a canonical basis, namely stably compact spaces. Specifically, we study the following question:

Question. If $A$ is a distributive-lattice-ordered algebra, how can sheaf representations of $A$ over a stably compact space $I$ be characterized dually?

We will report our current state of knowledge on this general question, and discuss the application of our work to MV-algebras. We will also briefly point to other algebraic structures that naturally arise in logic, to which similar methods could possibly be applied.