Proximity biframes and compactifications of completely regular ordered spaces

Guram Bezhanishvili and Patrick Morandi

The theory of compactifications of topological spaces and bispaces has been generalized to the point-free setting by Banaschewski and Schauerte. Compactifications of a frame (resp. biframe) are classified by strong inclusions on the frame (resp. biframe).

If (X, τ, \leq) is an ordered topological space in which the topology is strongly order convex, then this gives rise to a biframe (τ, τ_u, τ_d) , where τ_u (resp. τ_d) is the set of open upsets (resp. down sets) of X. Compactifications of (X, τ, \leq) were characterized by Nachbin. Such compactifications are more general than bicompactifications of the corresponding bispace.

In this talk we generalize Schauerte's notion of a compactification of a biframe and show that such compactifications correspond to Nachbin's compactifications in the spatial case. We introduce the notion of a proximity on a biframe, a related notion to that of Schauerte and Picado-Pultr, and show that proximities on a biframe are in 1-1 correspondence with compactifications of the biframe.