Modal logics of metric spaces
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Topological (c-)semantics, which interprets the modal diamond as closure in a topological space, was developed by McKinsey and Tarski in the 1930’s and 40’s. Their seminal result is that the modal logic of any dense-in-itself separable metric space is $\mathbf{S4}$. Utilizing the axiom of choice, Rasiowa and Sikorski strengthened this result by dropping the separability assumption. A related result of Bezhanishvili and Harding is that the modal logics of metric Stone spaces form the chain

$$\mathbf{S4.\text{Grz}_1} \supset \mathbf{S4.\text{Grz}_2} \supset \mathbf{S4.\text{Grz}_3} \supset \cdots \supset \mathbf{S4.\text{Grz}} \supset \mathbf{S4.1} \supset \mathbf{S4}.$$ 

We generalize these results by dropping the respective assumptions that the space is dense-in-itself, as well as, the space is compact and has a basis of clopen sets. Indeed, we show that the modal logics of metric spaces also form the above chain.