Modal logics of metric spaces

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joint work with Guram Bezhanishvili and David Gabelaia

Topological (c-)semantics, which interprets the modal diamond as closure in a topological space, was developed by McKinsey and Tarski in the 1930's and 40's. Their seminal result is that the modal logic of any dense-in-itself separable metric space is **S4**. Utilizing the axiom of choice, Rasiowa and Sikorski strengthened this result by dropping the separability assumption. A related result of Bezhanishvili and Harding is that the modal logics of metric Stone spaces form the chain

$$\mathbf{S4.Grz}_1 \supset \mathbf{S4.Grz}_2 \supset \mathbf{S4.Grz}_3 \supset \cdots \mathbf{S4.Grz} \supset \mathbf{S4.1} \supset \mathbf{S4}.$$

We generalize these results by dropping the respective assumptions that the space is dense-in-itself, as well as, the space is compact and has a basis of clopen sets. Indeed, we show that the modal logics of metric spaces also form the above chain.