Local homeomorphisms and Esakia duality

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The aim of this talk is to investigate the question: what is the correct analog for Esakia spaces of the notion of local homeomorphism of topological spaces, and how to characterize the corresponding homomorphisms of Heyting algebras?

For the case of finite Heyting algebras, respectively finite posets, we will give an answer in terms of the so-called *étale algebras* and *strict p-morphisms*.

The observation for the finite case can also help us to understand the general case.

We employ the duality between two three-step procedures:

- the well known Birkhoff procedure of generating a variety by a class of **base** algebras from products (P), subalgebras (S) and homomorphic images (h), and
- $\bullet^*$ gluing local homeomorphisms from disjoint unions ($\coprod = P^*$), open quotients ($Q = S^*$) and open subsets ($s = h^*$), starting from the **base** space of the local homeomorphism.

For a Heyting algebra H we thus arrive at the subvariety $\mathcal{V}_H$ of H-algebras generated by 1_H.

We will describe some particular cases and some general features of such varieties. In particular, in the finite case we will give an identity which characterizes the variety. In the general case, we will show that colimits in the variety $\mathcal{V}_H$ coincide with the colimits of the underlying lattices (in contrast with the variety of all algebras, where colimits are much more complicated).

As one possible direction towards applications, we will discuss connection with a problem of Andy Pitts and the work of the late Dito Pataraia on it.