A topological duality for filter-distributive congruential logics

Ramón Jansana

A logic has the congruence property when the mutual consequence relation between formulas is a congruence of the formula algebra. A logic is congruential if this property lifts to every algebra A in the sense that the relation that two objects have when they belong to the same logical filters of A is a congruence of A. A logic is filter-distributive when for every algebra A the lattice of its logical filters is distributive. Most of the well-known topological dualities for classes of algebras that correspond to a logic are for the classes of algebras that also correspond to a congruential and filter-distributive logic.

I will present joint work with María Esteban on a general framework to obtain topological dualities of a Priestley type for categories whose class of objects is the class of algebras that canonically corresponds to a filter-distributive congruential logic and whose morphisms are the algebraic homomorphisms.

For any filter-distributive congruential logic S, our main tools to obtain the duality for the category with objects the algebras in the algebraic counterpart Alg$S$ of S will be the notions of optimal logical filter and of S-prime strong logical ideal. Moreover, we associate with any algebra A in Alg$S$ the distributive meet-semilattice called the S-semilattice of A. The generalized Priestley space of this semilattice (as defined in Bezhanishvili, Jansana "Priestley style duality for distributive meet-semilattices" Studia Logica 98 (2011)) plays a crucial rôle in obtaining the dual space of A.