The modal logic of planar polygons

Kristina Gogoladze

joint work with David Gabelaia, Mamuka Jibladze, Evgeny Kuznetsov and Maarten Marx

We investigate the modal logic of the Euclidean plane in diamond-as-closure interpretation of modality when the valuations are restricted to the polygonal regions of the plane.

By a polygonal region we mean a finite Boolean (set-theoretic) combinations of bounded convex polygons. Since the planar polygonal regions turn out to be closed under the set-theoretic operations as well as the topological closure operator, it is possible to consider them as admissible sets in the generalized topological semantics of modal logic (akin to the general frame semantics). The collection of all valid modal formulas under such an interpretation over $\mathbb{R}^2$ (with valuations restricted to planar polygonal regions) is a normal extension of **S4** which we denote by $\mathbf{PL}_2$.

It is shown that $\mathbf{PL}_2$ is characterized by a certain class of finite frames of depth 3 which we call crown frames. The axiomatization of $\mathbf{PL}_2$ is provided by the technique of Jankov-Fine formulas. All the finite rooted non-frames of $\mathbf{PL}_2$ turn out to be subreducible to one of the five 'forbidden' finite frames, which allows the axiomatization. We also calculate the computational complexity of the satisfiability problem for $\mathbf{PL}_2$.

A similar approach of restricting attention to polygonal regions has been considered before in the setting of mereotopology. Our approach differs from those since we admit not only regular closed polygons, but also the regions of lower dimension (boundaries of polygons, segments and points), which might be relevant in specific applications of spatial KR&R.

Other precursors of this work that are more directly related to it in terms of the formalism employed are the considerations of serial subsets of the real line and the chequered subsets of the real plane from the modal-logical standpoint. Our work can be considered as an extension of these works. The logic $\mathbf{PL}_2$ we obtain is more complex however, since it is not tabular.