**Bitopological spaces with a nodec component and the same class of homeomorphisms**

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We recall that a topological space is nodec if its every nowhere dense subset is closed [5]. The class of nodec spaces, in particular, inclides submaximal, door, perfectly disconnected, maximal and I-spaces, modal logics of which are investigated in [2]. Besides interesting and important properties [5, 1], logical counterparts [2] and bitopological modifications [3, 4], it was found that nodec spaces can also be used for bitopological solution of one of Everett and Ulam's problems [6], namely – if $(X, \tau)$ is a given topological space and $\mathscr{H}(X, \tau)$ is the class of all homeomorphisms of $(X, \tau)$ onto itself, when and how a new topology $\gamma$ can be constructed on X such that $\mathscr{H}(X, \tau) = \mathscr{H}(X, \gamma)$?

By means of the original method proposed in [7] and the simple fact that in a nodec space without isolated points a subset is nowhere dense iff it is closed and discrete, we prove:

**Theorem.** *If $(X, \tau)$ is a nodec space without isolated points, then the family $\gamma = \{\varnothing\} \cup \{U \in \tau : X \setminus U$ is discrete$\}$ is a topology on X and $\mathscr{H}(X, \tau) = \mathscr{H}(X, \gamma)$ when every point of X has a neighbourhood in $(X, \tau)$ which is not dense in $(X, \gamma)$.*

## References


[1] A. V. Arhangelskii, P. J. Collins, *On submaximal spaces*. Topology and its Applications **64** (1995), 219–241.

[2] G. Bezhanishvili, L. Esakia, D. Gabelaia, *Modal logics of submaximal and nodec spaces*, in: "Collection of Essays Dedicated to Dick de Jongh on Occasion of His 65th Birthday", J. van Benthem, F. Veltman, A. Troelstra, A. Visser, editors. 2004, 713–732.

[3] B. Dvalishvili, *Connectedness of a fine topology and localization in bitopological spaces*. Georgian Math. J. **19** (2012), 449–472.

[4] B. Dvalishvili, *On various classes of bitopological spaces*. Georgian Math. J. **11** (2004), 713–732.

[5] E. K. van Douwen, *Applications of maximal topologies*. Topology and its Applications **51** (1993), 125–139.

[6] C. J. Everett, S. M. Ulam, *On the problem of determination of mathematical structures by their endomorphisms*. Bull. Amer. Math. Soc. **54** (1948), 646.

[7] Lee, Yu-lee, *Topologies with the same class of homeomorphisms*. Pacific Journal of Mathematics **20** (1967), 77–83.