Categories of equational theories

Marek Zawadowski
(joint work with Stanisław Szawiel)

Category of algebras of a (finitary) equational theory can be equivalently described as a category of models of a Lawvere theory, as a category of algebras of a finitary monad on the category Set, or as a categories of algebras of (generalized) operad in Set. Such a generalized operad can be thought of as a monoid in the monoidal category $Set^{\mathbb{F}}$ with substitution tensor ($\mathbb{F}$ is the skeleton of the category of finite sets). The correspondence of equational theories, Lawvere theories, monads and operads can be expressed as equivalence of four categories and it induces a correspondence between various subcategories of these categories. In some cases naturally defined subcategories of one of those four equivalent categories have natural and interesting characterizations as subcategories of other three categories. In some other cases naturally defined subcategories do not seem to have natural internal characterizations as subcategories of other categories.

In my talk, I will show examples of situations of both kinds. I will describe categories of Lawvere theories and equational theories that correspond to categories of analytic and polynomial monads. Then I will describe the categories of semi-analytic monads, regular Lawvere theories and regular operads that correspond to the category of regular equational theories. I will also show some examples where such correspondences seem to be less natural. Then I will consider some desirable properties of subcategories of equational theories and discuss which of those subcategories have them. This will explain, at least partially, why some correspondences considered earlier appear to be more natural than the others.