Duality theory in modal logic

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There are inherently two sides to the study of a logic: a syntactic and a semantic side. On the syntactic side, one describes a logic by specifying a set of axioms and derivation rules. This approach to logic is algebraic in nature. On the semantic side, one specifies a logic through an intended meaning or interpretation in some world. This approach is more geometric in nature. Understanding the connection between syntax and semantics is crucial in the study of a logic. Duality theory plays a central role in describing such connections. In this tutorial we start from classical proposition logic (CPL). Boolean algebras provide algebraic semantics for CPL (syntactic side). We describe how Stone's duality between Boolean algebras and Boolean spaces provides a bridge between the syntax and semantics of CPL. Thereafter we move to modal logic, which is an extension of CPL with additional operations (modalities) that may be used to express, for example, possibility, knowledge or provability. We will see that the basic Stone duality may be extended to a duality between modal algebras (which provide algebraic semantics for modal logic) and Kripke frames (which provide relational semantics for modal logic).