Guarded (co-)recursion, intuitionistic modal logics and scattered toposes

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Nakano's 2000 LiCS and 2001 TACS papers opened up the perspective of using Löb-style modalities a) for ensuring productivity of guarded (co-)recursion, b) in the (meta-) theory of logical step-indexed relations and c) in reactive programming. Much subsequent research has been done since in the area; it is enough to mention Appel et al. (POPL'07), Birkedal et al. (LiCS'11, POPL'11, FiCS'10), Krishnaswami&Benton (LiCS'11, ICFP'11), Krishnaswami et al. (POPL'12), Benton and Tabareau (TLDI'09) or Jaber et al. (LiCS'12).

In particular, the Copenhagen group has proposed and been actively studying a special class of toposes of (pre-)sheaves as a natural setting for such research, inspired by the Di Gianantonio and Miculan FoSSaCS 2004 work. The concrete example studied in the LiCS 2011 paper was the "topos of trees", i.e., presheaves on omega. What seems of utmost interest from the point of view of the ToLo III meeting is that this topos (and other relevant ones) can be seen as special cases of Esakia, Jibladze and Pataraia "scattered toposes" (APAL, 2000).

I will discuss the connection in more detail, in particular comparing the Tbilisi and the Copenhagen variants of topos-theoretical "internal fixed-point theorem". I will also discuss: the internal and external perspective on the Löb-like modalities in the topos setting; some of other results achieved so far by the Copenhagen group; a few open questions and future challenges.