Some modal logics arising from subspaces of the real line

Joel Lucero-Bryan

Topological d-semantics interprets the modal diamond as derivative in a topological space. Two important spaces are the real line ***R*** and its subspace of rational numbers ***Q***. Shehtman showed the d-logic of ***R*** is **K4DG2** and the d-logic of ***Q*** is **K4D**. A new proof of the latter result utilizes a geometric construction which yields a space that is homeomorphic to ***Q***. Upon prematurely terminating the construction, one obtains spaces that are homeomorphic to certain subspaces of ***Q***. From this construction we obtain that there are uncountably many d-logics arising from subspaces of ***Q***, uncountably many of which do not have the FMP and are neither finitely axiomatizable nor decidable. Among these d-logics are two descending chains of logics that expose new classes of topological spaces: semi-scattered and quasi-scattered spaces, each is a generalization of a scattered space. Furthermore, both the d-logic of all semi-scattered spaces and the d-logic of all quasi-scattered spaces are realizable, via the construction, as the d-logic of certain subspaces of ***Q***. Another generalization of a scattered space is known as a weakly scattered space. The construction can also be used to show the d-logic of all weakly scattered spaces is a d-logic of a subspace of ***Q***.