Canonical extension in first-order logic and Makkai's topos of types

Dion Coumans

Duality theory and its algebraic description via canonical extension have proven to be powerful tools in the study of propositional logics. This motivates the search for a generalisation of this theory to the setting of first-order logic. In this talk we define a notion of canonical extension for coherent categories. These are the categorical analogues of distributive lattices and they provide semantics for coherent logic (i.e., for the fragment of first-order logic in the connectives $\wedge$, $\vee$, 0, 1 and $\exists$). We show that our construction is characterised by a universal property and that it generalises the existing notion of canonical extension for distributive lattices. Furthermore, the construction restricts to the (non full) subcategory of Heyting categories, which provide semantics for intuitionistic first-order logic. Using our construction of canonical extension for coherent categories, we give an alternative description of the topos of types, which was introduced by Makkai in 1981. This allows us to give new and transparent proofs of some properties of the action of the topos of types construction on morphisms. We end by applying this description of Makkai's construction to relate, for a coherent category, its topos of types to its category of models (in **Set**).